Hydrodynamics of gravel-bed rivers: scale issues

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Abstract

The paper discusses several issues of gravel-bed river hydrodynamics where the scale of consideration is an inherent property. It focuses on two key interlinked topics: velocity spectra and hydrodynamic equations. The paper suggests that the currently used three-range spectral model for gravel-bed rivers can be further refined by adding an additional range, leading to a model that consists of four ranges of scales with different spectral behaviour. This spectral model may help in setting up scales of consideration in numerical and physical simulations as well as in better defining relevant fluid motions associated with turbulence-related phenomena such as sediment transport and flow-biota interactions. The model should be considered as a first approximation that needs further experimental support. Another discussed topic relates to the spatial averaging concept in hydraulics of gravel-bed flows that provides double-averaged (in time and in space) transport equations for fluid momentum (and higher statistical moments), passive substances, and suspended sediments. The paper provides several examples showing how the double-averaging methodology can improve description of gravel-bed flows. These include flow types and flow subdivision into specific layers, vertical distribution of the double-averaged velocity, and some consideration of fluid stresses.

Key words: gravel-bed rivers, turbulence, velocity spectra, fluid stresses, turbulence scales, flow types, velocity distribution.

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1. Introduction

The key feature that makes river flow different from other flow types is the interaction between flowing water and sedimentary bed. This interaction occurs over a wide range of scales, from the scale of a fine sediment particle to the basin scale. A small-scale subrange of this wide range of scales is formed by turbulence and turbulence-related processes. This subrange extends from sub-millimetres to a channel width and covers motion of sediment particles in individual and collective (bedforms) modes, mixing and transport of various substances (e.g., nutrients, contaminants) and flow-biota interactions. These turbulence-related phenomena are especially important in the functioning of gravel-bed rivers and therefore constantly attract researchers’ attention, consistently forming a topic of discussion at Gravel-Bed Rivers Workshops (e.g., Livesey et al., 1998; Nelson et al., 2001; Roy and Buffin-Belanger, 2001; Wilcock, 2001).

At present, turbulence research of gravel-bed rivers is based on two fundamental physical concepts: eddy/energy cascade and coherent structures. Originally these concepts have been developed independently, and it is only recently that researchers started viewing them as interlinked phenomena. These concepts, together with fundamental conservation equations for momentum, energy, and substances, represent two facets of flow dynamics: statistical and
deterministic. The deterministic approach stems from some ‘coherency’ in turbulent motions and from hydrodynamic equations based on fundamental conservation principles, while the statistical approach recognises ‘irregular’ components in hydrodynamic fields and therefore focuses on their statistical properties. The statistical approach is based on two important procedures: (1) decomposition of hydrodynamic fields into slow (or mean) and fast (or turbulent) components; and (2) averaging or filtering of instantaneous variables and corresponding hydrodynamic equations. The first procedure is known as the Reynolds decomposition in the case of time and ensemble (i.e., probabilistic) averaging and as Gray’s (1975) decomposition in the case of spatial averaging. This procedure can be interpreted as a scale decomposition or separation of scales. The second procedure can be formulated in many different ways among which time, ensemble, and area/volume averaging are most common. This second procedure can be viewed as a scaling-up procedure that changes the scale of consideration from one level in time-space-probability domain to another level. In this respect, scale is an inherent feature of any hydrodynamic equation, which is not always recognised in Earth Sciences. The generalised hydrodynamic equations formulated in terms of statistical moments of various orders were first proposed by A.A. Friedman and L.V. Keller in the 1920s (Monin and Yaglom, 1971). As an example, the well-known Reynolds averaged Navier-Stokes equation represents an equation for the first-order moments of velocity and pressure fields. Another direction within the statistical approach is formulation of statistical turbulence theories based on physical intuition rather than on basic conservation principles expressed by hydrodynamic equations. A well known example is Kolmogorov’s turbulence theory and its associated “-5/3” law for the inertial subrange where energy is transferred from larger scales to smaller scales without dissipation and/or additional production. Scale is an inherent feature in such theories as well. It can be argued that the currently popular terms in Earth Sciences such as scaling, scale-invariance, self-similarity, characteristic scales, and scaling behaviour largely stem from these statistical theories of turbulence (e.g., Barenblatt, 1995, 2003).

The range of problems and concepts related to gravel-bed river turbulence is wide and it is impossible to address them all within a single paper. Instead, following the central theme of this Workshop, the paper will review topics where the scale issue, as describe above, is a fundamental feature, proper account of which may improve current understanding of gravel-bed rivers dynamics. The velocity spectra in gravel-bed rivers will be discussed first as it forms a general framework for multi-scale considerations. With this as background, a brief discussion on how time and spatial scales are associated with currently used hydrodynamic equations will follow. This will lead to a more detailed consideration of the double-averaging methodology dealing with hydrodynamic equations averaged in both time and space. In the author’s own research, this methodology evolved in the mid 1990s when he tried to use conventional Reynolds averaged equations to study near-bed region of gravel-bed flows and found them inconvenient because of scale inconsistency. The paper will conclude with several examples showing how the double-averaging methodology can improve description of gravel-bed flows. The examples include flow types and flow subdivision into specific layers, vertical distribution of the double-averaged velocity, and some consideration of fluid stresses. The examples support a view that this methodology opens a new perspective in gravel-bed rivers research and may help in clarifying some long-standing problems.

There are many other important aspects of gravel-bed river turbulence that are not covered in this paper. Interested readers will benefit from checking a comprehensive report of Lopez and Garcia (1996) and very recent reviews of the problem given in Roy et al. (2004, and references therein) and Lamarre and Roy (2005, and references therein).
2. Velocity spectra in gravel-bed rivers

Velocity fluctuations in gravel-bed rivers cover wide ranges of temporal and spatial scales, from milliseconds to many years and from sub-millimetres to tens of kilometres. The smallest temporal and spatial scales relate to the so-called dissipative eddies through which energy dissipation occurs due to viscosity. The largest temporal scales of velocity fluctuations relate to long-term (climatic) fluctuations of the flow rate, while the largest spatial fluctuations are forced by morphological features such as meanders or even larger structures of, for example, tectonic origin. The amplitude of velocity fluctuations typically increases with period and wavelength (i.e., with the scale). This dependence can be conveniently summarised using velocity spectra showing how the energy of fluctuations is distributed across the scales (Fig. 1). The spectra in Fig. 1 represent a result of conceptualisation of extensive turbulence and hydrometric measurements (Grinvald and Nikora, 1988). The low frequency (large periods) range in the frequency spectrum is formed by intra-annual and inter-annual hydrological variability while high-frequency (small periods) range is formed by flow turbulence (Fig. 1a). The connection between these two extreme ranges is not yet clear and may relate to various large-scale flow instabilities (Grinvald and Nikora, 1988), defined in Fig. 1a as “hydraulic phenomena”. The low wave-number (large spatial scale) range in the wave-number spectrum is formed by morphological variability along the flow such as bars and/or meanders (Fig. 1b), as was mentioned above. At small spatial scales (less than flow width) velocity fluctuations are due to turbulence. If the wave-number and frequency turbulence spectra can often be linked through Taylor’s ‘frozen’ turbulence hypothesis (as can be seen in Figs. 1a and 1b, Nikora and Goring, 2000a), the relationship between large-scale ranges of the wave-number and frequency spectra are not as clear. The turbulence ranges in Figs. 1a and 1b can be conceptually subdivided into macro-turbulence (between flow depth and flow width), meso-turbulence (between dissipative scale and flow depth), and micro-turbulence (dissipative eddies). There may be a variety of energy sources for flow turbulence with the key source being the energy of the mean flow, which is transferred into turbulent energy through velocity shear and through flow separation behind multi-scale roughness elements. In the velocity spectra, the first transfer occurs at the scale of the flow depth while the second transfer occurs at the scale of roughness size(s) Δ (Fig. 1a and 1b). The importance of a particular range in turbulence dynamics and specific boundaries of spectral ranges should depend on width to depth ratio and relative submergence.

The information on turbulence spectra in gravel-bed rivers is very fragmentary and mainly covers the longitudinal velocity component \( u \) in the range of scales from approximately one tenth of depth to several depths (e.g., Grinvald and Nikora, 1988; Nezu and Nakagawa, 1993; Roy et al., 2004). It has been shown that this region usually covers the inertial subrange where velocity spectra follow Kolmogorov’s “-5/3” law. In most such studies a three-range model of spectra has been accepted, implicitly or explicitly, which consists of: (1) the production range where spectral behaviour has not been identified specifically; (2) the inertial subrange where spectra follow the “-5/3” law (there is no energy production or dissipation in this subrange, Monin and Yaglom, 1975); and (3) the viscous range where spectral density decays much faster than in the inertial subrange. This conceptual model stems from Kolmogorov’s concept of developed turbulence (i.e., at sufficiently large Reynolds number, Monin and Yaglom, 1975). However, the true spectral behaviour outside the range of length-scales from \( \approx 0.1 \) to \( \approx (2 \text{ to } 3) \) flow depths, although very important for engineering and ecological applications, is not yet clear. Below this range of scales is revised and extended using physical and scaling arguments, and then compared with available measurements.
The analysis begins with the reasonable assumption that velocity spectra $S_y(k)$ in high Reynolds number gravel-bed flows with dynamically completely rough beds are fully determined by one velocity scale (i.e., the shear velocity $u_\ast$), and three characteristic length-scales: (1) characteristic bed particle size (or roughness length) $\Delta$, assuming that it essentially captures the effects of bed particle size distribution; (2) distance from the bed $z$ (see section 4.3 for a discussion of bed origin); and (3) mean flow depth $H$. These are the main scales for flows over both fixed and mobile beds. The bed conditions for our considerations are somewhat simplified, i.e., channel width and characteristic scales of bed-forms are excluded from our analysis. Also, the viscous range of scales is not considered. This range, although important for dissipation mechanisms (which are beyond the scope of this paper), contributes very little to the total spectral energy. With these assumptions one can have:

$$S_y(k) = F(u_\ast, \Delta, z, H, k)$$  \hspace{1cm} (1)

where $k$ is longitudinal wave number in the direction of the mean flow ($k = 2\pi / \lambda$, $\lambda$ is an eddy characteristic scale in the streamwise direction). After applying conventional dimensional analysis relationship (1) reduces to:

$$S_y(k) = u_\ast^2 k^{-1} f(k\Delta, kz, kH)$$  \hspace{1cm} (2)

Using (2) one may consider spectral behaviour of (i) the largest eddies ($\lambda > a_1 H$), (ii) intermediate eddies ($a_2 z < \lambda < a_1 H$), and (iii) relatively small eddies ($a_3 \Delta < \lambda < a_2 z$) where $a_{1i}, a_{2i}$, and $a_{3i}$ are scaling coefficients for the $i$-th velocity component ($i=1$ for the longitudinal component $u$, $i=2$ for the transverse component $v$, and $i=3$ for the vertical component $w$).

For the largest eddies ($\lambda > a_1 H > a_2 z > a_3 \Delta$), i.e.:

$$kH < 2\pi b_{1i} < 2\pi b_{2i} \frac{H}{z} < 2\pi b_{3i} \frac{H}{\Delta} \quad \text{or} \quad kz < 2\pi b_{1i} \frac{z}{H} < 2\pi b_{2i} < 2\pi b_{3i} \frac{z}{\Delta}, \quad \text{where} \quad b_{ki} = \frac{1}{a_{ki}}$$  \hspace{1cm} (3)

incomplete self-similarity in $kH$ (or self-similarity of the second kind after Barenblatt, 1995, 2003), and complete self-similarity in $k\Delta$ and $kz$ (note that $k\Delta < kz < kH$) are assumed. The latter means that at small $k\Delta$ and $kz$ the spectrum does not depend on these variables and they can be dropped while the former means that at a small ratio $H / \lambda \propto kH$ we may present $S_y(k) = f(kH)$ as $S_y(k) \propto (kH)^\alpha$. In the case of the complete similarity in $kz$ the contributions to spectra from the largest eddies are invariant with respect to distance from the bed, which seems physically reasonable (e.g., Kirkbride and Ferguson, 1995; Nikora and Goring, 2000b; Roy et al., 2004; Nikora, 2005). All these reduce (2) to:

$$S_y(k) = c_{ij} u_\ast^2 k^{-1} (kH)^\alpha$$  \hspace{1cm} (4)

where $c_{ij}$ is a constant. Relationship (4) can be further simplified using a physical argument that the largest eddies represent a link between the mean flow and turbulence, i.e., in spectra they occupy the region of turbulence energy production where eddies interact with the mean
flow and between themselves. This energy exchange between large eddies suggests that their spectral contributions are invariant with wave number and so $k$ should be dropped from (4).

This assumption gives $\alpha = 1$ and simplifies relationship (4) to a form:

$$S_y(k) = c_{iy} u_i^2 H \quad \text{or} \quad S_y(kH) = c_{iy} u_i^2 \quad \text{or} \quad S_y(kz) = c_{iy} u_i^2 \left( \frac{z}{H} \right)^{-1} \quad (5)$$

which is valid for $kz < 2\pi b_{1}/(z/H)$ (see (3)).

For the intermediate eddies ($a_{2i}z < \lambda < a_{ii}H$), i.e.:

$$\frac{2\pi b_{1i}}{H} < kz < 2\pi b_{2i} < 2\pi b_{3i} \frac{z}{\Delta} \quad (6)$$

complete self-similarity with $k\Delta$, $kz$, and $kH$ is assumed that reduces (2) to the relationship:

$$S_y(k) = c_{2yi} u_i^2 k^{-1} \quad \text{or} \quad S_y(kH) = c_{2yi} u_i^2 (kH)^{-1} \quad \text{or} \quad S_y(kz) = c_{2yi} u_i^2 (kz)^{-1} \quad (7)$$

which is valid for $2\pi b_{1i}/(z/H) < kz < 2\pi b_{2i}$ and where $c_{2yi}$ is a constant.

Relationships (6) and (7) mean that eddies from this range of scales are independent of the characteristic scales $\Delta$, $z$, and $H$ and depend only on the velocity scale, i.e., $u_*$. 

Finally, for relatively small eddies ($a_{3i}\Delta < \lambda < a_{2i}z$), i.e.:

$$\frac{2\pi b_{1i}}{H} < 2\pi b_{2i} < kz < 2\pi b_{3i} \frac{z}{\Delta} \quad (8)$$

incomplete self-similarity with $kz$ and complete self-similarity with $k\Delta$ and $kH$ are assumed, i.e.:

$$S_y(k) = c_{3yi} u_i^2 k^{-1} (kz)^{\beta} \quad (9)$$

where $c_{3yi}$ is a constant. To define an exponent $\beta$ one may use a reasonable assumption that these eddies form the inertial subrange, i.e., $S_y(k) \propto k^{-5/3}$ and $C_{uw}(k) \propto k^{-7/3}$ (Monin and Yaglom, 1975) where $S_y(k)$ is the auto-spectrum for the $i$-th velocity component (i.e., the spectrum of a single velocity component), and $C_{uw}(k)$ is the co-spectrum, which is the real part of the cross-spectrum of longitudinal and vertical velocities. The analysis here is restricted to just this one off-diagonal component of the spectral tensor since $C_{uw}(k)$ provides important information on contributions from different eddies to the primary shear stress $-u'w'$, i.e.,

$$\overline{u'w'} = \int_{-\infty}^{\infty} C_{uw}(k) dk \quad.$$ This assumption gives $\beta = -2/3$ for the auto-spectra and $\beta = -4/3$ for the co-spectra and simplifies (9) to the following relationships:
\[ S_u(k) = c_{3u} u_*^2 k^{-5/3} z^{-2/3} \quad \text{or} \quad S_u(kz) = c_{3u} u_*^2 (kz)^{-5/3} \] (10)

\[ C_{uw}(k) = c_{3uw} u_*^2 k^{-7/3} z^{-4/3} \quad \text{or} \quad C_{uw}(kz) = c_{3uw} u_*^2 (kz)^{-7/3} \] (11)

which are valid for \(2\pi b_{2l} < kz < 2\pi b_{3l} z / \Delta\). Note that the distance \(z\) from the bed in (8) may be interpreted as an ‘external’ Kolmogorov’s scale (Monin and Yaglom, 1975) defined by the size of ‘attached’ eddies (i.e., eddies ‘growing’ from the bed; the attached eddies hypothesis was first introduced by Townsend, 1976). This suggests that \(k_{23}z = \text{const}\) where \(k_{23}\) is the low-wave-number limit for the inertial subrange, i.e., the boundary between (7) and (10). The performance of the above relationships for individual velocity components, wave-number limits for (5), (7), (10), and (11) and ‘universal’ constants \(c_{1yy}, c_{2yy}\), and \(c_{3yy}\) should be defined from experiments.

The above conceptual model consists of four ranges of scales with different spectral behaviour: (I) the range of the largest eddies (\(\lambda > a_i H\)) with \(S_y(k, z) \propto k^0 z^0\); (II) the range of intermediate eddies (\(a_i z < \lambda < a_i H\)) with \(S_y(k, z) \propto k^{-1} z^0\); (III) the range of relatively small eddies (\(a_i \Delta < \lambda < a_i z\)) with \(S_y(k, z) \propto k^{-5/3} z^{-2/3}\) and \(C_{uw}(k, z) \propto k^{-7/3} z^{-4/3}\) (known as the inertial subrange where no energy production or dissipation occurs); and (IV) the viscous range (not specified here). In addition to previous three-range concepts for open-channel flows (e.g., Grinvald and Nikora, 1988; Nezu and Nakagawa, 1993) this model specifies the spectral behaviour at very low wave numbers and adds an additional spectral range with \(S_y(k) \propto k^{-1}\) (Fig. 2a).

If spectral ranges (I), (III), and (IV) are well known and are widely used in physical considerations, range (II) with \(S_y(k) \sim k^{-1}\) is much less known in gravel-bed rivers research. Its physical origin is still unclear (see, e.g., Yaglom, 1993; Katul and Chu, 1998 for various concepts and associated references). A plausible physical mechanism that may explain the appearance of this spectral range is briefly reviewed below, following Nikora (1999). There are two important properties of wall turbulence (close to the bed, within the logarithmic layer which is assumed to exist) which are well tested and accepted in wall turbulence studies.

A. The shear stress \(\tau\) is approximately constant and equal to \(\tau = \rho u_*^2\) \((u_*\) is the friction velocity, and \(\rho\) is fluid density). The production of the total turbulence energy \(P\) is approximately equal to the energy dissipation \(\varepsilon_d\) that leads to \(P \approx \varepsilon_d \sim u_*^3 / z\). These properties describe Townsend’s (1976) equilibrium wall layer with constant shear stress.

B. The mean flow instability and velocity shear generate a hierarchy of eddies attached (in the sense of Townsend, 1976) to the bed so that their characteristic scales are proportional to the distance \(z\) from the bed.

Using property B it can reasonably be assumed that, due to flow instability and velocity shear, the energy injection from the mean flow into turbulence occurs at each distance \(z\) from the wall, with generation of eddies with characteristic scale \(L \sim z\). These eddies transfer their energy at rate \(\varepsilon\) to smaller eddies and may be viewed as energy cascade initiators. In other
words, it is suggested here that at each \( z \) a separate Kolmogorov’s cascade is initiated which is superposed with other energy cascades initiated at other \( z \)’s. As a result of this superposition of cascades, the energy dissipation \( \varepsilon_d \) at a particular distance \( z \) presents a superposition of down-scale energy fluxes, \( \varepsilon \), generated at this and at larger \( z \) (contribution from cascades generated at smaller \( z \) is negligible; justification for this may be found in Townsend, 1976). Thus, the energy flux \( \varepsilon \) across the scales at any \( z \) depends on the scale under consideration, i.e., on wave number \( k \). The flux \( \varepsilon \) increases with \( k \) until it reaches \( 2\pi z \) and then, for \( k \geq (2\pi / z) \), stabilises being equal to \( \varepsilon_d \) (Fig. 2a). In other words, at a given distance \( z_g \) the energy flux \( \varepsilon \) for \( k < 2\pi z_g^{-1} \) represents the energy dissipation \( \varepsilon_d \) observed at \( z = 2\pi k^{-1} \), \( z > z_g \). Using property A (i.e., \( \varepsilon_d \sim u_i^2 / z \)) and bearing in mind that \( L \sim z \sim k^{-1} \), we have \( \varepsilon(k) \sim u_i^2 k \) for \((2\pi / H) \leq k \leq (2\pi / z_g) \). The scale \( H \) is an external scale of the flow. Following this phenomenological concept and using the inertial subrange relationships \( S_{u_i}(k) \sim \varepsilon_d^{2/3} k^{-5/3} \) and \( C_{uu}(k) \sim \varepsilon_d^{4/3} u_i^{-2} k^{-7/3} \) one can obtain (7), (10) and (11). Thus, the existence of the “-1” spectral law in wall-bounded turbulence is explained by the effect of superposition of Kolmogorov’s energy cascades generated at all possible distances from the wall, within an equilibrium layer. This concept is justified using only the well-known properties of wall-bounded flows. The energy cascades initiated at any \( z \) may be linked to large eddies attached to the bed and scaled with \( z \). Such eddies may be associated with coherent structures, considered for example in Roy et al. (2004). Indeed, the data presented in Nikora (2005) suggest that the clusters of bursting events are the main contributors to range I with \( S_y(k) \propto u_i^2 H \) while range II with \( S_y(k) \propto u_i^2 k^{-1} \) is probably formed by individual bursting events. The latter may be viewed as the energy cascade initiators.

The four-range model described above has been well supported by data from gravel-bed flows (e.g., Nikora and Smart, 1997; Nikora and Goring 2000b; Roy et al., 2004). As an illustration, Fig. 2b shows normalised spectra \( S_y(kz)/u_i^2 \) for three representative values of \( z \) (so that effects of normalisation can be clearly seen without attenuation by numerous curves) measured with Acoustic Doppler Velocimeters in a gravel-bed Balmoral Canal (New Zealand). It is evident from Fig. 2b that the measured spectra do support (7) and (10) for all three velocity components, although the “-1” ranges for the transverse and vertical velocities are fairly narrow. Besides, this figure also supports scaling relationships (7) and (11) for the co-spectra. At low wave numbers all spectra tend to constant values as predicted by (5). The typical values for the constants \( c_{ij}, c_{2y}, \) and \( c_{3y} \) obtained for gravel-bed flows are \( c_{1uy} \approx 1.0, c_{1vv} \approx 0.13, c_{1uw} = 0.04 \) [see eq. (5)], \( c_{2uw} = 0.90, c_{2vy} = 0.50, c_{2uw} = 0.30 \) [see eq. (7)], and \( c_{3uw} = 0.90, c_{3vy} = 1.20, c_{3uw} \approx 0.9 \) [see eq. (10)]. The standard measurement errors of the above values are within 5-25%. Note that the ratio \( c_{3wy} / c_{3uw} \) does not satisfy Kolmogorov’s theory of locally-isotropic turbulence, i.e., \( c_{3wy} / c_{3uw} \approx 1.0 < 4 / 3 \), which is probably due to deviation from local isotropy.

The satisfactory agreement between the proposed scaling model and measurements for the intermediate flow region (not very close to either the bed or water surface) may have immediate applications for the broad-band turbulence intensities. Indeed, the integration of the total spectra for velocity components \( u, v, \) and \( w \) gives:
\[
\left( \frac{\sigma_i}{u_*} \right)^2 = M_i - N_i \ln\left( \frac{z}{H} \right) \quad \text{and} \quad \frac{K}{u_*} = 1.84 - 1.02 \ln\left( \frac{z}{H} \right) \quad (12)
\]

where \( \sigma_i \) is the standard deviation of \( i \)-th velocity component; \( K = 0.5(\sigma_u^2 + \sigma_v^2 + \sigma_w^2) \) is the total turbulent energy; and \( M_u = 1.90, \quad M_v = 1.19, \quad M_w = 0.59, \quad N_u = 1.32, \quad N_v = 0.49, \quad \text{and} \quad N_w = 0.22 \) are derived from field experiments (Nikora and Goring, 2000b). Equations (12) show how the turbulence intensity and energy change with changing distance from the bed.

Although the above conceptual model for velocity spectra in gravel-bed flows is plausible and well supported by data for particular hydraulic conditions, it should be treated as a preliminary result rather than a solution of the problem. Indeed, the model, while applicable for some conditions, has many limitations and does not cover many other possible scenarios encountered in the field. The future improvements should account for the effects of relative submergence, width-to-depth ratio, multi-scale bed forms, wake turbulence, aquatic vegetation, bed permeability, topology of coherent structures, and other factors.

3. Scales, hydrodynamic equations, and the double-averaging methodology

Velocity fluctuations in gravel-bed rivers, highlighted in the previous sections, form a wide continues spectrum that makes statistical approach for their description and prediction inevitable. Although ideally it would be preferable to study the whole range of scales simultaneously (i.e., resolving the smallest temporal and spatial scales involved), in practical terms it is impossible and often is not necessary. In principle, the small-scale effects can be incorporated into larger-scale dynamics by integrating corresponding hydrodynamic equations. This procedure, as already mentioned, is commonly formulated as either time or ensemble or area/volume averaging, or combination of them. This scaling-up procedure is in-built into currently used hydrodynamic equations. Indeed, depending on temporal and spatial resolution these equations can be broadly classified as: (1) equations with no time/ensemble and spatial averaging for (instantaneous) hydrodynamic variables (e.g., Navier-Stokes equation for momentum, NS); (2) spatially-filtered hydrodynamic equations for variables with small-scale spatial averaging (e.g., Large Eddy Simulation, LES; no time averaging is involved); and (3) time-(or ensemble) averaged hydrodynamic equations with no spatial averaging, known as the Reynolds Averaged NS equations (RANS). The spatial (LES) or time (RANS) averaging of NS equations for instantaneous variables can be viewed as a scaling-up procedure that changes the scale of consideration from a point in time-space (as in NS) to a larger spatial (as in LES) or temporal (as in RANS) scales. This classification can be further extended by adding hydrodynamic equations for variables averaged in both time and space, which can be defined as the double-averaged hydrodynamic equations (DANS). The double averaging up-scales the original NS in both time and space domains. The selection of the equations for hydraulic modelling is often based, implicitly or explicitly, on scale considerations, i.e., bearing in mind velocity spectra considered in the previous section.

To take all advantages provided by direct numerical solution (DNS) of the Navier-Stokes equations or LES one needs access to high-performance computing facilities as well as highly resolved initial and boundary conditions, which are unlikely to be available for many real-life engineering or ecological applications. Therefore, the RANS-based modelling approach is currently the most popular in solving practical problems although methodologically it is inconsistent in accounting for drag forces acting on the rough bed. In many real-life situations the commonly used RANS equations are difficult to implement due to the highly three-
dimensional small-scale structure of the mean flow and turbulence, especially in the near-bed region (e.g., Lamarre and Roy, 2005). In addition, most applications deal with spatially-averaged roughness parameters that cannot be linked explicitly with local (point) flow properties provided by the Reynolds equations. A more straightforward approach is to use the DANS-based models, which are rigorously derived for rough-bed flows and provide explicit guidance in closure development and parameterizations. DANS-based models may successfully fill a gap in modelling capabilities for gravel-bed flow problems where the RANS-based models are not suitable. In the next paragraph we introduce the double-averaging methodology which potentially may advance current understanding of gravel-bed flow hydrodynamics as well as provide a practicable modelling tool.

The double-averaged equations for turbulent rough-bed flows have been first introduced and advanced by atmospheric scientists dealing with air flows within and above terrestrial canopies such as forests or bushes (Wilson and Shaw, 1977; Raupach and Shaw, 1982; Finnigan, 1985, 2000). Later the double-averaging approach has been adopted in environmental hydraulics (e.g., Gimenez-Curto and Corniero Lera, 1996; McLean at al., 1999; Lopez and Garcia, 2001; Nikora et al., 2001, 2004, 2006a, 2006b) but its applications for modelling, experimental design, and data interpretation are still largely undeveloped. This section will briefly discuss the double-averaged momentum equation that then will be used to illustrate the advantages of this methodology. Similar equations can be also derived for conservation of mass, energy, and other velocity moments (e.g., turbulent shear stresses). In Nikora et al. (2006a) it has been shown that for a reasonably general case of static and mobile bed surfaces with roughness elements such as moving gravel particles the double-averaged (in time first and in space second) momentum equation can be written as:

\[
\frac{\partial \langle \vec{u}_i \rangle}{\partial t} + \langle \vec{u}_i \rangle \frac{\partial \langle \vec{u} \rangle}{\partial x_j} = g_i - \frac{1}{\rho \phi} \frac{\partial \langle p \rangle}{\partial x_i} - \frac{1}{\phi} \frac{\partial \langle \vec{u}_i \vec{u}_j \rangle}{\partial x_j} + \frac{1}{\phi} \frac{\partial}{\partial x_j} \left( \rho \frac{\partial \vec{u}_i}{\partial x_j} \right) + \frac{1}{\rho \phi} \frac{1}{V_o} \int \int_{S_{in}} \frac{1}{S_{nt}} \int \int \vec{u} \frac{\partial u_i}{\partial x_j} \cdot n_j dS
\]

where \( u_i \) is the \( i \)-th component of the velocity vector; \( p \) is pressure; \( g_i \) is the \( i \)-th component of the gravity acceleration; \( \rho \) is fluid density; \( V_o \) is the total volume of the averaging domain (thin slab parallel to the mean bed); \( n \) is the inwardly-directed unit vector normal to the bed surface (into the fluid); \( S_{nt} \) is the extent of water-bed interface bounded by the averaging domain; and \( \phi \) is the rough bed ‘porosity’ also defined in Nikora et al. (2001) as the roughness geometry function; it is discussed in the next paragraph. An overbar and angle brackets in equation (13) denote time and spatial (volume) averaging, respectively. The superscript “s” denotes superficial time average (Nikora et al. 2006a) when averaging time interval includes both periods when the spatial points are intermittently occupied by fluid and when they are occupied by roughness elements (e.g., by moving gravel particles). Equation (13) uses Reynolds’ decomposition \( \theta = \bar{\theta} + \theta' \) for instantaneous variables and an analogue of Gray’s (1975) decomposition for the time-averaged variables, \( \bar{\theta} = \langle \bar{\theta} \rangle + \bar{\theta} \), where \( \theta \) is a hydrodynamic variable. The wavy overbar denotes the spatial fluctuation in the time-averaged flow variable, i.e., the difference between the double-averaged \( \langle \bar{\theta} \rangle \) and time-averaged \( \bar{\theta} \) values (\( \bar{\theta} = \bar{\theta} - \langle \bar{\theta} \rangle, \langle \bar{\theta} \rangle = 0 \)), similar to the conventional Reynolds decomposition of \( \theta = \bar{\theta} + \theta' \),
\( \bar{\theta} = 0 \). The spatial averaging is often performed over a volume \( V_o \) that is a thin slab parallel to the mean (or ‘smoothed’) bed. The plane dimensions of the averaging domain should be larger than typical mean flow heterogeneities, introduced by roughness, but much smaller than the large-scale features in bed topography. For gravel-bed rivers they should be much larger than gravel particles, but much smaller than sizes of riffles or pools.

Equation (13) has been derived in a single-step procedure from the Navier-Stokes equation for instantaneous variables using the averaging theorems linking double-averaged derivatives with derivatives of the double-averaged variables (Nikora et al., 2006a). It accounts for roughness mobility and change in roughness density with spatial coordinates and with time, which make them different from similar equations considered in terrestrial canopy aerodynamics and porous media hydrodynamics. The derivation of (13) accounted for both spatial porosity \( \phi_s = V_f / V_o \) and ‘time’ porosity \( \phi_t = T_f / T \), where \( V_f \) is the volume occupied by fluid within an averaging (total) volume \( V_o \); \( T \) is the total averaging time interval including periods when the spatial points are intermittently occupied by fluid and roughness elements (e.g., by moving gravel particles); and \( T_f \) is the averaging time interval equal to sum of time periods when a spatial point under consideration is occupied by fluid only. In equation (13) it is assumed that \( \phi_t \) does not (spatially) correlate with the time-averaged flow parameters (i.e., \( \langle \phi_t \bar{\theta} \rangle = \langle \phi_t \rangle \langle \bar{\theta} \rangle \)).

For many applications this assumption is reasonable and allows replacing the product \( \phi_t \bar{\theta} \) with a single symbol \( \phi = \phi_s \phi_t \). For fixed (static) roughness elements we have \( \phi_t = T_f / T = 1 \) and thus \( \phi = \phi_s \). For gravel-bed flows, the function \( \phi(z) \) changes upwards from the bed material porosity \( \phi_{\text{min}} \) deeply in the sediment layer to 1 at the roughness tops to zero at the air-water interface. In the case of a flat water surface, there is a discontinuity in \( \phi(z) \) when it changes from 1 to 0. In the case of a disturbed water surface (e.g., random surface waves), the change in \( \phi(z) \) from 1 to 0 is likely to be smooth, similar to the water-sediment interface. Note that in previous work (e.g., Nikora et al. 2001), the roughness geometry function was defined for area averaging and denoted by a symbol \( A \). Here we use the symbol \( \phi \) to make distinction between area and volume averaging.

In comparison with the conventional Reynolds-Averaged Navier-Stokes equation, the proposed double-averaged momentum equation contains several additional terms which explicitly present dispersive or form-induced stresses \( \langle \bar{u}_i \bar{u}_j \rangle \) due to spatial variations in time-averaged fields, the form drag per unit fluid volume \( f_{\text{fr}} = 1 / (\phi V_o) \int_{S_{\text{ex}}} \rho n dS \), and viscous drag per unit fluid volume \( f_{\text{visc}} = -1 / (\phi V_o) \int_{S_{\text{ex}}} \rho v \partial u_i / \partial x_j n dS \). The quantities \( \langle \bar{u}_i \bar{u}_j \rangle \) in equation (13) stem from spatial averaging, similar to \( \bar{u}_i \bar{u}_j \) in the Reynolds-Averaged equations which represent a result of time (ensemble) averaging of the Navier-Stokes equation for instantaneous variables. In other words, the double-averaged equations relate to the time-averaged equations in a similar way as the time-averaged equations relate to the equations for instantaneous hydrodynamic variables. Assessment of the significance of these terms in (13) for different hydrodynamic and bed roughness conditions is currently underway (e.g., Nikora et al., 2006b). An important additional advantage of using double-averaged hydrodynamic parameters and equations is a better coupling between the surface water flow and the sub-surface flow within the porous bed where volume-averaged variables are traditionally used (e.g., Whitaker, 1999).
The next section provides a brief review of several issues of gravel-bed flow dynamics which are discussed based on the double-averaging methodology and which illustrate its advantages.

4. Hydrodynamics of gravel-bed flows: double-averaging perspective

4.1. Vertical structure of gravel bed flows

Based on an analysis of the double-averaged momentum equation (13), Nikora et al. (2001, 2006b) suggested four types of rough-bed flows (Fig. 3), depending on flow submergence $H_m/\Delta$ ($H_m$ is the maximum flow depth, i.e., the distance between water surface and roughness troughs). Here this classification is adopted, with some modifications, for gravel-bed flows.

The flow type I is the flow with high relative submergence, which contains several layers and sublayers (neglecting viscous sublayers associated with gravel particles): (1) near-water-surface layer where flow structure is influenced by the free surface effects such as turbulence damping and various types of water surface instabilities, and which for a dynamic non-flat air-water interface may be further subdivided into an upper sub-layer with a smooth transition in $\phi(z)$ from 1 (water) to 0 (air) where drag terms and $\langle \ddot{u} \ddot{u}_j \rangle$ in (13) may be important, and a lower sublayer where form-induced stresses $\langle \ddot{u} \ddot{u}_j \rangle$ may be essential (these sublayers are similar to the interfacial and form-induced sublayers at water-sediment interface described below); (2) outer or intermediate layer, where viscous effects and form-induced momentum fluxes due to water surface disturbances and bed roughness are negligible, and the spatially-averaged equations are identical to the time-averaged equations; (3) the logarithmic layer (as the relative submergence is large enough to form an overlap region) that differs from the outer layer by characteristic velocity and length scales; (4) the form-induced (or dispersive) sublayer, below the logarithmic layer and just above the roughness crests, where the time-averaged flow may be influenced by individual roughness elements and thus the terms $\langle \ddot{u} \ddot{u}_j \rangle$ may become non-zero; (5) the interfacial sublayer, which occupies the flow region between roughness crests and troughs and where momentum sink due to skin friction and form drag occurs; and (6) subsurface layer below the interfacial sublayer. The interfacial and form-induced sublayers, combined together, can be defined as the roughness layer. The near-water-surface layer can be viewed as a near-surface counterpart of the roughness layer. The other three flow types are: (II) flow with intermediate relative submergence consisting of the subsurface layer, a roughness layer, an upper flow region which does not manifest a genuine universal logarithmic velocity profile as the ratio $H_m/\Delta$ is not large enough, and the near-water-surface layer; (III) flow with small relative submergence with a roughness layer overlapped with the near-water-surface layer; and (IV) flow over a partially-inundated rough bed consisting of the interfacial sublayer overlapped with the near-water-surface layer (Fig. 3). These four flow types and their subdivision into specific layers are based on the presence and/or significance of the terms of equation (13) in a particular flow region and cover the whole range of possible flow submergence $H_m/\Delta$.

The above flow subdivision and flow types represent a useful schematization that may help in various problems of gravel-bed flows. For each flow type, a specific set of relationships describing ‘double-averaged’ flow properties may be developed. The next section addresses the vertical distribution of the double-averaged longitudinal velocity $<\ddot{u}>$, partly based on Nikora et al. (2004).
4.2. Velocity distribution

Vertical distributions of the time-averaged velocity in gravel-bed flows can be highly variable within a reach and therefore their parameterisation and prediction may be achievable only for flows with high relative submergence and away from the bed where local effects of roughness elements are not felt. Another difficulty arises from the fact that most applications deal with spatially-averaged roughness parameters that cannot be linked explicitly with local time-averaged velocities, which are variable in space. An alternative approach is to use the double-averaged velocities instead. Their vertical distribution should depend on the flow type and, furthermore, within a particular flow type it may differ in shape from layer to layer (Fig. 3). This section first briefly considers the least studied flow region defined in Fig. 3 as the interfacial sublayer, and then provides some discussion on the velocity distribution above the gravel tops.

Considering the simplest case of two-dimensional, steady, uniform, spatially-averaged flow over a fixed rough bed, Nikora et al. (2004) derived several models applicable to a range of flow conditions and roughness types that share some common features. Two of these models for the interfacial sublayer (linear and exponential) are directly applicable to gravel bed flows. The exponential model applies when the effect of the momentum flux downwards dominates over the gravity term in (13) that leads to the exponential velocity distribution:

\[ \langle \overline{u} \rangle (z) = \langle \overline{u} \rangle (z_c) \exp (\beta (z - z_c)) \]  

(14)

where \( \langle \overline{u} \rangle (z_c) \) is the double-averaged velocity at the roughness crests \( z_c \); and \( \beta \) is a parameter. The linear model may be a good approximation for gravel beds where the function \( \phi \) monotonically decreases while the total drag term \((f_p + f_r)\) monotonically increases towards the lower boundary of the interfacial sublayer (Nikora et al., 2001), i.e.:

\[ \frac{\langle \overline{u} \rangle (z) - \langle \overline{u} \rangle (z_c)}{u_*} = \frac{(z - z_c)}{l_c} \]  

(15)

where \( l_c = \langle \overline{u} \rangle (z_c) / (d\langle \overline{u} \rangle / dz) \) is the shear length scale characterizing flow dynamics within the roughness layer; and \( \delta \) is the thickness of the interfacial sublayer. In principle, relationships (14) and (15) may be applicable for the interfacial sublayer for all four types of gravel-bed flows defined above, from flows with large relative submergence to flows with partial submergence. Fig. 4 supports this conjecture by showing examples of vertical distributions of the double-averaged velocity obtained in laboratory experiments for flow types I, II, and III (high to small relative submergence) and covering roughness types with various densities and arrangements (Nikora et al. 2004). In real gravel-bed rivers, the double-averaged velocity profiles within the interfacial sublayer are expected to be more complicated and composed of a combination of the linear and exponential models.

The distribution of the double-averaged velocity above the roughness layer for flow type I (with large relative submergence) follows the logarithmic formula (e.g., Nikora et al., 2001):

\[ \frac{\langle \overline{u} \rangle}{u_*} = \frac{1}{\kappa} \ln \left[ \frac{z - d}{\delta_R} \right] + C = \frac{1}{\kappa} \ln \left[ \frac{z - d}{z_o} \right] \text{ for } z \geq z_R \]  

(16)
where $\kappa$ is the von Karman constant; $\delta_r$ is the thickness of the roughness layer; $d$ is the displacement length (also known as a zero-plane displacement) that defines the ‘hydrodynamic’ bed origin; $z_o = \delta_r \exp(-\kappa C)$ is the hydrodynamic roughness length; and the constant $C$ depends on the definition of $\delta_r$ and the roughness geometry (see Fig. 3 for definitions).

It is useful to recall that equation (16) has been phenomenologically justified only for flows with large relative submergence where roughness scale $\Delta$ is well separated from the external flow scale such as mean flow depth $H$. For the genuine universal logarithmic layer (16) to form the required ratio $H/\Delta$ should well exceed 40 or even 80 (Jimenez, 2004). However, many gravel-bed flows can often be defined as flows with intermediate submergence (flow type II), i.e., they are relatively shallow with respect to the multi-scale bed roughness ($H/\Delta$). With no alternative rigorous theory available these low-submergence flows are nearly always studied using the logarithmic boundary layer concept, which is currently justified only for deep flows, i.e., $H/\Delta$ > 80 (Jimenez, 2004). Nevertheless, the data available for flow type II suggest that the shape of the distribution of the double-averaged velocities above the roughness layer is often logarithmic (e.g., Bayazit, 1976; Dittrich and Koll, 1997; Dancey et al., 2003) and, therefore, there might be some general law behind it. Below an explanation for logarithmic behaviour in flows of type II is suggested by modifying an overlap-based derivation of the logarithmic formula.

Following the conventional dimensional analysis we can express the vertical distribution of the double-averaged velocity in the near-bed region above the roughness layer as:

$$\frac{\langle u \rangle}{u_*} = F\left(\frac{z-d}{\Delta}, \frac{H}{\Delta}, \gamma_i\right)$$  \hspace{1cm} (17)

where $\gamma_i$ are the dimensionless parameters of bed roughness (e.g., density of roughness elements); and the flow depth is defined as the difference between the mean water surface elevation and the mean bed elevation. Formula (17) represents the inner layer where roughness effects on the velocity field dominate. In the flow region well away from the bed, the velocity deficit $\langle u \rangle - \langle u \rangle_{\text{max}}$ can be expressed as:

$$\frac{\langle u \rangle - \langle u \rangle_{\text{max}}}{u_*} = G\left(\frac{z-d}{H}, \frac{H}{\Delta}\right)$$  \hspace{1cm} (18)

where $\langle u \rangle_{\text{max}}$ is the maximum flow velocity at $z = z_o$ that often occurs at the water surface. Formula (18) represents the outer layer where effects of large eddies scaled with the flow depth dominate. Equations (17) and (18) differ from conventional relationships (e.g., Raupach et al., 1991) by additional variable $H/\Delta$ included in both functions $F$ and $G$. At very large relative submergence $H/\Delta$ it is reasonable to assume that functions $F$ and $G$ do not depend on $H/\Delta$. This case corresponds to conventional formulation (16) for flows with large relative submergence. However, at smaller values of $H/\Delta$ (flow type II) equations (17) and (18) state that effects of large eddies on the inner layer are not negligible with, at the same time, bed roughness effects extending into the outer layer. For this case, we can assume that there is an overlap region between the inner and outer layers where equations (17) and (18) are
simultaneously valid, similar to the classical overlap approach. Then, equating the derivatives of (17) and (18) and multiplying them by \((z-d)\) we can obtain:

\[
\frac{z-d}{u_*} \frac{\partial \langle \bar{u} \rangle}{\partial z} = Z_A \frac{\partial F}{\partial Z_A} = Z_H \frac{\partial G}{\partial Z_H} = f\left(\frac{H}{\Delta}\right) = \left[\kappa\left(\frac{H}{\Delta}\right)\right]^{-1}
\]

where \(Z_A = (z-d)/\Delta\), \(Z_H = (z-d)/H\), and the function \(f\) depends on the relative submergence \(H/\Delta\) as it is present as a variable in both \(F\) and \(G\). This function is expressed here as \(f(H/\Delta) = \left[\kappa(H/\Delta)\right]^{-1}\) for convenience, preserving classical formulation but interpreting \(\kappa\) as the von Karman parameter rather than a constant. From equation (19) it is clear that when \(H/\Delta\) is not large enough then the von Karman parameter \(\kappa\) depends on \(H/\Delta\). Integration of (19) gives:

\[
\frac{\langle \bar{u} \rangle}{u_*} = \frac{1}{\kappa(H/\Delta)} \ln\left[\frac{z-d}{\Delta}\right] + C\left(\frac{H}{\Delta}, r_i\right)
\]

and

\[
\frac{\langle \bar{u} \rangle - \langle \bar{u} \rangle_{\text{max}}}{u_*} = \frac{1}{\kappa(H/\Delta)} \ln\left[\frac{z-d}{z_m-d}\right]
\]

Equations (20) and (21) can serve, at least as a first approximation, for describing velocity distribution above roughness tops in flows with intermediate submergence (flow type \(\text{II}\)). The data available for such flows (e.g., Bayazit, 1976; Dittrich and Koll, 1997; Dancey et al., 2003) support equations (20) and (21) and show that with increase in relative submergence the von Karman parameter \(\kappa\) tends to the well-known universal constant of 0.41. Although relationships (20) and (21) are to be yet properly tested they represent a useful framework for interpreting and explaining experimental data on flows of type \(\text{II}\).

4.3. Bed origin and zero-plane displacement

Equations (16) – (21) contain a displacement height \(d\) that determines the origin of the logarithmic velocity profile. It is useful therefore to discuss what bed elevation should be used as the bed origin for hydrodynamic considerations. In general, at least three different 'hydrodynamic' bed origins may be distinguished, depending on the particular task (Nikora et al., 2002). The bed origin of type 1 corresponds to the level that should be used to measure the flow depth and the bed shear stress. It can be readily shown, using the spatial averaging approach (Nikora et al., 2001), that the spatially averaged bed elevation should be used as the bed origin in this case. The size of the spatial averaging area, which is in the plane parallel to the mean bed and which is the same for both bed topography and the hydrodynamic variables, depends on the statistical structure of bed elevations, i.e., on their probability distribution and the spectrum or correlation functions. The bed origin of type 1 is a natural choice when one considers 2D vertically-averaged hydrodynamic equations (models). The definition of this bed origin does not require any details of velocity distribution as it is based purely on bulk mass conservation and spatially-averaged momentum balance. Although the bed origin of this type is equally useful in 3D considerations too, there are also other options for this case. One of them is the bed origin for the logarithmic formula of the velocity distribution, defined here as the bed origin of type 2. Another useful bed origin, type 3, may be defined for the spatially-averaged
velocity distribution within the roughness layer. Two natural choices for this bed origin may be considered, i.e.: (i) the minimum elevation of the roughness troughs, and (ii) the upper boundary of the interfacial sublayer as expressed in equations (14) and (15). Both lower and upper boundaries of the interfacial sublayer may be defined in a statistical sense (e.g., 5% and 95% probability of exceedence). Bed origins of types 1, 2, and 3 do not necessarily coincide as sometimes assumed in research papers. They also do not exclude each other as they represent different flow features and, thus, all of them are useful in modeling and physical considerations. Justification for the bed origin types 1 and 3 is clear and reasonably straightforward. However, the bed origin of type 2 for the logarithmic formula is still under debate (see, e.g., Nikora et al., 2002 for review). In Nikora et al. (2002) it was suggested that the zero-plane displacement \(d\) for the logarithmic formula should be the level that large-scale turbulent eddies feel as the 'bed' and, thus, their dimensions linearly scale with the distance from this virtual bed. Such a definition directly follows from a slightly modified Prandtl's mixing length phenomenology, and serves as a physical basis for determining \(d\) from velocity measurements. Nikora et al. (2002) demonstrated that for a range of roughness types the displacement height for the logarithmic formula is strongly correlated with the thickness of the interfacial sublayer and with the shear length scale \(l_c = \langle \bar{u} \rangle (z_r)/(d\langle \bar{u} \rangle / dz)_{z_c}\) in (15).

4.4. Fluid stresses in gravel-bed flows

Within the double-averaging framework, the total fluid stress in gravel-bed flows consists of three components:

\[
\tau(z) = \rho \left\{ \frac{1}{\phi} \nu \frac{d\phi\langle \bar{u} \rangle}{dz} - \langle \bar{u}'w' \rangle - \langle \bar{\nu}\bar{w} \rangle \right\}
\]

which are expressed in (22), for simplicity, for the case of 2D flows. In most cases the viscous component in (22) can be neglected, as turbulent stresses in gravel-bed flows are normally several orders of magnitude larger. The spatially-averaged turbulent stress \(-\langle u'w' \rangle\) often changes (quasi) linearly towards the bed and attains a maximum near the roughness tops. Below the roughness tops it reduces to zero due to momentum sink through viscous (skin) friction and form drag. Its distribution is reasonably well studied experimentally showing dependence on relative submersion and roughness geometry. The last component, form-induced stress \(-\langle \bar{\nu}\bar{w} \rangle\), appears as a result of spatial correlation of perturbations in time-averaged velocities leading to an additional ‘canal’ of momentum flux. The change in flow submersion may lead, in principle, to the change in nature of fluid stresses. Gimenez-Curto and Corniero Lera (1996) suggested that with a decrease in flow submersion, the form-induced stress might become the dominant component of the total stress. This, in turn, may lead to a new flow regime named by the authors the “jet regime”, in addition to the well-known laminar, turbulent hydraulically-smooth, and turbulent hydraulically-rough regimes (Gimenez-Curto and Corniero Lera, 1996). However, the nature of form-induced stresses in gravel-bed flows is still unclear. Some preliminary experimental results suggest that \(-\langle \bar{\nu}\bar{w} \rangle\) may not be negligible within the roughness layer and their contributions and role in the momentum balance can be important (up to 15-30% of the total fluid stress, Nikora et al., 2001; Aberle and Koll, 2004; Campbell et al., 2005).

5. Conclusions
In this paper, several issues of gravel-bed river hydrodynamics were discussed, with the focus on two key interlinked topics: velocity spectra and hydrodynamic equations, related to each other through the scale of consideration. It is suggested that the currently used three-range spectral model for gravel-bed rivers should be further refined by adding an additional range, leading to a model that consists of four ranges of scales with different spectral behaviour. This model is considered as a first approximation that needs further experimental support. Another topic relates to the spatial averaging concept in hydraulics of gravel-bed flows that provides double-averaged transport equations for fluid momentum (and higher statistical moments), passive substances, and suspended sediments. The double-averaged hydrodynamic equations considered in this paper may help in developing numerical models, designing and interpreting laboratory, field, and numerical experiments.

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References


Figure Captions for the paper

“Hydrodynamics of gravel-bed rivers: scale issues”

by V. Nikora

Fig. 1. Schematised velocity spectra in gravel-bed rivers: (a) frequency spectrum; and (b) wave-number spectrum ($W_o$ and $W$ are the river valley and river channel widths, respectively).

Fig. 2. (a) Schematised velocity auto-spectra $S_u(kz)$ and energy transfer rate $\varepsilon(kz)$ showing:
1. the large-scale energy production range ($k > H^{-1}$); (2) the “-1” scaling range ($H^{-1} < k < z^{-1}$) where energy cascades initiated at each $z$ are superimposed and $\varepsilon(k)$ changes as $\varepsilon(k) \sim k$; (3) the inertial subrange ($k > z^{-1}$) which results from superposition of inertial subranges generated at each $z$ and, therefore, $\varepsilon(k) = \varepsilon_d$; and (4) the dissipative range; (b) An example of velocity spectra at $z/H=0.0095$, 0.0380, and 0.4670, measurements were made with acoustic Doppler velocimeters (ADV) with the sampling frequency of 25 Hz and duration of 20 min, 1997, Balmoral Canal (flow rate = 5.14 m$^3$/s; cross-sectional mean velocity = 1.05 m/s; cross-sectional mean depth = 0.78 m), New Zealand [see also Nikora (2005) for more details].

Fig. 3. Flow types and flow subdivision into specific regions in gravel-bed flows.

Fig. 4. Vertical distribution of the double-averaged velocities for various roughness types in coordinates $[(\langle \bar{u} \rangle(z) - \langle \bar{u} \rangle(z_f))]/u_* = f[(z - z_f)/l_f]$ (data are fully described in Nikora et al., 2004). Deviations of the data points from $Y=X$ are consistent with the exponential distribution (14).
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