Water Balance Modelling in Alpine Catchments at Different Spatial and Temporal Scales

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Notation

**Time scale**
- \( [\bar{X}] \) Long term mean annual value of \( X \).
- \( \langle \bar{X} \rangle \) Long term mean monthly value of \( X \).
- \( [X] \) Annual value of \( X \).
- \( \langle X \rangle \) Monthly value of \( X \).
- \( X \) No brackets: daily value of \( X \).
- \( \hat{A} \) Fuzzy number of variable or parameter \( A \).
- \( \mu \) Level of presumption.

**(+)** Symbol of fuzzy addition; all fuzzy arithmetic operations are symbolised with brackets, e.g.: \((+), (\cdot), (\div), (\equiv), (>), (\preceq)\) and \((\neq)\).

- \( a^l, a^c, a^r \) Characteristic left, centre and right values, respectively, of a triangular fuzzy number \( \hat{A} \).
- \( [a^l(\alpha), a^c(\alpha)] \) \( a^l(\alpha) \) and \( a^c(\alpha) \) indicate the lower and upper bounds of the interval \( \hat{A}(\alpha) \) at any \( \alpha \)-level in the interval \([0,1]\).
- \( \Delta^l(\alpha) \) Interval between \( a^l(\alpha) \) and \( a^c \) at any \( \alpha \in [0,1] \); same for \( \Delta^r(\alpha) \).
- \( \Delta t \) Time interval: year, month or day which is explicitly stated in the text [1].

**Topography**
- \( H \) Elevation; \( H_{\text{zone}} \) denotes a mean elevation estimate of one elevation zone [m a.s.l.].
- \( f_{H_{\text{zone}}} \) Fraction of catchment area associated with elevation zone \( H_{\text{zone}} \) [1].

**Soil**
- \( \phi \) Soil porosity [1].
- \( \theta_{\text{wp}} \) Permanent wilting point [1].
- \( \theta_{fc} \) Field capacity [1].
- \( pc \) Percentage from total area covered by respective soil type [%].
- \( I \) Number of different soil types [1].
- \( D_{\text{sp}} \) Total depth of the soil profile to an impervious layer [mm].
- \( C_{\text{sp}} \) Total soil moisture capacity of the soil profile above the permanent wilting point. The maximum storage capacity of the bucket model [mm].
- \( C_{fc} \) Soil moisture capacity until field capacity of the soil profile [mm].
- \( r_s \) Soil moisture ratio; \( S \) is a fraction of \( C_{fc} \) [1].
Storage level in the runoff control store. $S_{sr}$, $S_{in}$ and $S_{bf}$ refer to the storage modules simulating surface runoff, interflow and baseflow, respectively [mm].

Snow

$Q_N$ Snowmelt runoff at thawing conditions; during rain events ($Q_N$, wet) or during non-rain periods ($Q_N$, dry) [mm/\(\Delta t\)].

$NM$ Snowmelt due to the radiate heat transfer during precipitation events [mm/\(\Delta t\)].

$EM$ Snowmelt due to the latent heat transfer during precipitation events [mm/\(\Delta t\)].

$HM$ Snowmelt due to the sensible heat transfer during precipitation events [mm/\(\Delta t\)].

$RM$ Snowmelt due to rain [mm/\(\Delta t\)].

$R_{ex}$ Extraterrestrial short wave radiation heat received on a sloping surface on a certain day of the year [J/m²].

$e_{ann}$ Coefficient to correct for absorption and scattering of short wave radiation in the atmosphere [1].

$e_{clo}$ Coefficient to correct for absorption and reflection of short wave radiation by clouds [1].

$CC$ Long term average of the seasonally varying cloud coverage [1].

$e_{cor}$ Coefficient to correct for the contribution of scattered light to net short wave radiation [1].

$e_{veg}$ Coefficient to account for the portion of light transmitting through the vegetation cover [1].

$e_{alb}$ Coefficient to correct for the reflection of short wave radiation at the snow surface [1].

$fc$ Cover density of the predominant vegetation above the snowpack, which is assumed to be equal to the forest-cover density [1].

$\varepsilon_{air}$ Average emissivity of air [1].

$R_{air}$ Average long wave radiation heat emitted from a perfect black body at air temperature on a certain day of the year [J/m²].

$R_{snow}$ Average long wave radiation heat emitted from the snowpack (perfect black body) at the snow surface temperature (0 °C) on a certain day of the year [J/m²].

$mf$ Melt factor for snowmelt processes at thaw conditions. $mf_{\text{min}}$ is assumed to occur on December 21st and $mf_{\text{max}}$ on June 21st [mm/K/\(\Delta t\)].

$T_{0}$ Threshold air temperature between frost or thaw situations within $\Delta t$ [°C].

$T_{t}$ Transition mean air temperature within $\Delta t$. In the case of a single $T_{t}$ snowfall below $T_{t}$ and rain above $T_{t}$. Otherwise pure snowfall below $T_{t,\text{min}}$, pure rain above $T_{t,\text{max}}$, mixed snow and rain events between $T_{t,\text{min}}$ and $T_{t,\text{max}}$ [°C].

$T_{\text{crit}}$ Critical mean air temperature: threshold between frost or thaw situations (threshold air temperature $T_{0}$) and rain or snow events (transition air temperature $T_{t}$) [°C].

$T_{pos}$ Mean air temperature within $\Delta t$. If $T_{pos}$ is below $T_{0}$ then $T_{pos}$ is set to $T_{0}$ [°C].

$H_{pos}$ Sum of mean air temperature intervals above the threshold mean air temperature $T_{\text{crit}}$ [K].
**Notation**

$H_{neg}$
Sum of mean air temperature intervals below the threshold mean air temperature $T_{crit}$ [K].

c
Number of temperature classes in the histogram of normal distributed mean daily temperature per month [1].

$n_j$
Number of days belonging to class $j$ [1].

$T_j$
Mean daily temperature belonging to class $j$ [°C].

$N$
Number of days per month [1].

$J$
Number of days per year [1].

$S_N$
Storage of snow water equivalent in the snowpack [mm].

**Meteorology**

$P$
Precipitation [mm/$\Delta$t].

$P_r$
Precipitation falling as rain [mm/$\Delta$t].

$P_s$
Precipitation falling as snow [mm/$\Delta$t].

$T$
Mean air temperature within $\Delta t$, e.g. $\langle T_i \rangle$ assigns the long term mean monthly temperature estimates at monitoring sites and $T_{crit}$ symbolises the daily mean catchment estimates of temperature [°C].

$t_{st}$
Mean length of storm event [days].

$t_{i-st}$
Mean length of inter-storm period [days].

$m$
Number of storms [1].

$t_{wet}$
Number of days with precipitation (precipitation larger than 0.5 [mm/d]) [days].

$t_{dry}$
Number of days without precipitation (precipitation equal or less than 0.5 [mm/d]) [days].

$E$
Evapotranspiration: Potential ($E_p$) or actual evapotranspiration ($E_a$) [mm/$\Delta$t].

$DI$
Index of dryness ($\langle E_p \rangle / \langle P \rangle$) [1].

**Discharge**

$Q$
Discharge at closure section: observed ($Q_o$) and modelled discharge ($Q_p$) [mm/$\Delta$t].

$Q_{sr}$
Surface runoff [mm/$\Delta$t].

$Q_{se}$
Saturation excess runoff [mm/$\Delta$t].

$Q_{ss}$
Sub-surface runoff [mm/$\Delta$t].

$Q_{in}$
Interflow [mm/$\Delta$t].

$Q_{bf}$
Baseflow [mm/$\Delta$t].

$Q_l$
Lateral flow component [mm/$\Delta$t].

$Q_{p-in}$
Percolation into the storage module [mm/$\Delta$t].

$Q_{p-out}$
Percolation out from the storage module [mm/$\Delta$t].

$Q_{sr-in}$
Percolating surface runoff to deeper interflow zone [mm/$\Delta$t].

$Q_{in-bf}$
Percolation from interflow to baseflow zone [mm/$\Delta$t].
Notation

$t_c$  
Catchment response time of $Q_{ss}$ [day].

$t_{c,in}$  
Catchment response time of $Q_{in}$ [day].

$t_{c,bf}$  
Catchment response time of $Q_{bf}$ [day].

$t_{c,ar}$  
Catchment response time of $Q_{ar}$ [day].

$t_{c,ar-in}$  
Response time of $Q_{ar-in}$ [day].

$t_{c,in-bf}$  
Response time of $Q_{in-bf}$ [day].

$h$  
Threshold storage level before $Q$ starts [mm/Δt].

Statistical Disaggregation in Space and Time

$X'$  
Transformed value of $X$ (Equation 7.1, Schröder 1969): $X' = f(X_{min}, X_{max}, C)$.

$C$  
Coefficient in transformation according to Schröder (1969) [1].

$r$  
Coefficient of correlation; e.g. $r_{T-H,space}^{12}$ signifies the coefficients of correlation of the spatial distributions of mean monthly temperature and elevation within the catchment for all months and $r_{P-T, time}^{12}$ denotes the coefficients of correlation of catchment daily precipitation and temperature within all months [1].

$CV$  
Coefficient of variation; e.g. $CV_{P, space-time}$ indicates the coefficients of variation of the distribution of daily precipitation within months and within the Gail catchment [1].

$CS$  
Coefficient of skewness [1].

$\sigma_X$  
Standard deviation of $X$ [1].

$a$ and $b$  
Coefficients of linear regression function ($y = ax + b$), e.g. between observed mean monthly temperature ($y = \{\bar{T}_{m} \}$) and elevation ($x = H$) [1].

$frq(H)$  
Frequency of elevation $H$ [1].

$f(X)$  
Probability of variables $X$ [1].

$F(X)$  
Cumulative frequency of variables $X$; e.g. $F(P_{cm})$ indicates the cumulative frequency of catchment daily precipitation ($P_{cm}$) within a month [1].

$f(X,Y,r)$  
Bivariate probability as function of variable $X$, variable $Y$ and the coefficient of correlation $r$; e.g. $f(P_{cm}, T_{cm}, r)$ indicates the probability of transformed daily mean catchment precipitation ($P_{cm}$) and daily mean catchment temperature ($T_{cm}$) within a month [1].

$a$  
Value associated with iso-line (e.g. 0.99, 0.9, 0.5 and 0.1) assigned in bivariate probability density function [1].

$pdf$  
Probability density function: $pndf$ assigns the probability normal density function and $bpndf$ indicates the bivariate probability normal density function.

$cdf$  
Cumulative distribution function: $cnf$ assigns the cumulative normal distribution function and $bcnf$ indicates the bivariate cumulative normal distribution function.

Water balance model performance criterion

$MD$  
Sum of the mean differences between simulated and observed discharge [1].
Notation

\[ \text{NS} \quad \text{Nash-Sutcliffe criterion (Nash and Sutcliffe 1970) [1].} \]

\[ \text{CM} \quad \text{Chiew-McMahon criterion (Chiew and McMahon 1994) [1].} \]
Chapter 1: Introduction

This research is addressed towards the general problem of predicting the water balance in ungauged catchments. Specifically, it focuses on the problem of extracting appropriate model structures by systematic analysis of rainfall-runoff relationships in gauged catchments. The Upper Enns and the Gail catchments in the Austrian Alps are selected as the basis of this study.

Scientific Objectives
− To identify the characteristic hydro-meteorological features that emerge at different spatial (catchment, sub-catchment and sub-regional scale) and temporal scales (annual, monthly and daily time scale).
− To present the downward approach to model conceptualisation, based on emergent hydrological, meteorological and physiographic properties of the water balance with changing time scales.
− To develop parsimonious annual, monthly and daily water balance models with a good trade-off between model performance and model complexity, aiming for a small set of model parameters.
− To present the formulation of a lumped water balance model in the framework of fuzzy logic.
− To demonstrate the ability of a fuzzy model to quantify the relative importance of various parameters and input values.
− To demonstrate the interconnections between model complexity, predictive uncertainty, and accuracy of predictions.
− To identify the appropriate spatial scales for simulation of the semi-distributed monthly water balance components of a typical alpine catchment.
− To define the spatial and temporal correlation of distributed temperature and precipitation which have the most impact on the monthly water balance.

Outline of Thesis
In Chapter 2, the downward approach is followed, which involves stepwise adjustment of model structure to capture the observed streamflow variability progressively at the annual, monthly, and then on to daily time scales. Throughout it is focused on emergent properties of the hydrological system at the various time scales, as detected in key signature plots and hydrographs, and model complexity is always kept to the minimum required. Any further alteration or calibration of parameter values is avoided either with change of scales or in response to inadequate predictions. The downward approach presented leads to parsimonious water balance models with excellent performance and the minimum set of parameters, with a good balance being achieved between model performance and complexity.

Chapter 3 presents the application of a fuzzy water balance model to the Upper Enns catchment in the Austrian Alps and demonstrates its ability to deal with uncertain and imprecise information. The core water balance model used is a parsimonious model developed previously in Chapter 2. This existing model is recast into a fuzzy framework so as to incorporate uncertain, and qualitative information on physical and meteorological catchment characteristics, as well as experiential information of a quantitative character. All
variables and parameters in the model are expressed in terms of fuzzy membership functions to account for the uncertainties involved in the estimation of model parameter values and climatic inputs. Computations with the resulting fuzzy water balance model lead to fuzzy model outputs that are characterised by intervals of confidence at a chosen “level of presumption”. This fuzzy water balance model is used further to carry out sensitivity analyses with respect to model parameter values and climatic inputs, as well as model structural complexity. The results give insights into the relative importance of various parameters, processes and inputs, and shed light on the question of how much complexity is needed in water balance models for specific applications of the model. The results can be used to target our efforts towards improvement of parameter and input estimation procedures and the formalisation of parsimonious models for catchment water balance with a focus on predictive uncertainty. Models based on fuzzy parameterisation are potentially suitable for predictions of ungauged catchments because of their natural ability to deal with uncertain, imprecise and experiential information.

Chapter 4 addresses the need for distributed water balance estimates in alpine catchments. Such estimates are an essential prerequisite for efficient water resources planning and management. Spatial disaggregation into sub-regions of similar climatic and topographic properties is an optimal method for considering all stages of water balance estimation: input data generation, model parameterisation, and water balance simulation. Various techniques for the spatial interpolation of precipitation and temperature point observations are evaluated in terms of their applicability to mountain terrain. A water balance model is introduced for the computation of seasonal and regional distributed water balance for an alpine catchment. The conceptual model is characterised by minimal model complexity, most of its parameters being estimated a priori from catchment physiography. In this manner, an automatic model calibration is avoided. The concept of water balance estimation is applied to the transboundary Gail river catchment located between Austria and Italy, south of the main alpine divide.

Chapter 5 focuses on the prediction of the monthly water balance based on monthly climate data in alpine regions with boreal climate conditions. Generally, models perform insufficient when attempting to compute the monthly water balance with lumped mean monthly climate data. Hydro-meteorological properties that emerge at smaller spatial scales than the catchment scale and smaller temporal scales than the monthly scale are important determinants of the monthly water balance. The spatial and temporal within month distribution of temperature and precipitation data is analysed and analytical distribution and regression functions are parameterised. Several monthly water balance models are formulated on various different combinations of spatial disaggregation of mean catchment data with temporal distribution of monthly mean climate input data. At all stages of model conceptualisation the model structures are kept parsimonious through hypothesis testing thereby evaluating the model performance with a special focus on the number of applied model parameters. The model performances are evaluated through annual and monthly signature plots and hydrographs as well as with performance measures. The number of applied model parameters ranges between six and fifteen, although excellent model performance is already achieved with several parsimonious models using twelve parameters. The Gail catchment is selected as a case study catchment because it is influenced by continental, Mediterranean and Adriatic weather systems. Hence, results should also be applicable for other alpine catchments with less hydro-meteorological complexity.
Chapter 2: Modelling Water Balances in Alpine Catchment through Exploitation of Emergent Properties over Changing Time Scales

Keywords:
- annual, monthly and daily water balance
- hydrologic modelling
- downward approach
- emergent properties
- model structure
- Upper Enns catchment

2.1 Introduction

Prediction of streamflow, using information on climatic inputs and catchment attributes only, requires a greater level of understanding of climate, soil, vegetation and topographic controls on catchment water balance than we have at present. Our current understanding of climate, soil, vegetation and topographic controls of water balance, such as that incorporated in Eagleson (1978), has been based on theories developed at the small (hydrodynamic or laboratory) scale, such as Darcy’s law or the Manning or Chezy equation, and underlain by assumptions of homogeneity. Such small-scale theories cannot be easily extended to make predictions at larger scales, such as annual runoff predictions at catchment scales (Klemes 1983), because of the enormous heterogeneity and variability exhibited in natural catchments and the associated data and computational requirements. It is now well recognised that as one goes over changes of scale, both in time and in space, dominant hydrological processes may exhibit different modes of transitions, either sharp and/or gradual changes. For example, vertical vadose zone processes or macropore influences dominate at small plot scales, whereas topography begins to dominate runoff processes at the hillslope scale, and the stream network may begin to dominate catchment organisation, spatial soil moisture variations, and patterns of runoff generation at the catchment scale, as schematically illustrated in Blöschl and Sivapalan (1995). A similar situation occurs when one goes through time scale changes, say from the diurnal scale to the event scale and then to the annual time scale. Conceptual structures of models capturing the dominant processes at one set of time and space scales may be adequate for a partial range of time and space domains around these scales, but may lose their predictability at much larger scales (Klemes 1983). It is not clear if models based on the small-scale theories and concepts we are familiar with (Richards equation, St. Venant equations etc.) can identify and deal with such emergent properties well beyond the scales at which they were developed. In contrast to traditional physically-based models based on the small-scale theories, such as the SHE model (Abbott et al. 1986), so-called conceptual models (see Franchini and Pacciani 1991; Singh 1995) have the advantage that 1) they are not necessarily based on small-scale physical theories, and 2) since many conceptual models are based on systems theoretic approaches, they have the potential to provide a more holistic representation of catchment response (Holzmann et al. 1998). However, their disadvantage is that model structures are often chosen a priori without taking advantage of insightful knowledge
of the particular hydrological situation that may be embedded in observations. Indeed, gauged streamflows are
often used to calibrate a preconceived model rather than to gain insightful knowledge of the particular catchment
with a view to developing appropriate model structures. In other words, there is no concept of learning from the
data. There is as yet no accepted methodology for applying a priori system knowledge gained from data analysis
for the selection or development of physically based, water balance models.

Outline of Chapter
This chapter deals with the systematic analysis of precipitation and streamflow data, for making inferences about
catchment properties that emerge with changing time scales, with a view to gaining insightful knowledge of
catchment behaviour, and aimed at the development of parsimonious model structures. The approach presented
here, in many respects, parallels the data-based mechanistic modelling approach pioneered by Young and his
colleagues over the past decade (Young 1998; see also Young 2001, for a list of key references), and differs from
it in the fact that model structures are chosen by means of intuitive, more physically-based and hydrological
arguments. In the approach presented here, it is started with a simple model structure at the annual time scale,
and then the evolution of model structure and complexity is investigated with decreasing time scales, from
annual to monthly to daily, and then the evolving model structures are linked to the emergent properties of
catchment response. This downward approach has been championed by Klemes (1983). This approach has been
previously utilised by Jothityangkoon et al. (2001) and Farmer et al. (2002) to investigate appropriate model
structures for a number of semi-arid catchments in Australia. This chapter differs from these in that it is focused
on a humid, Alpine catchment.

The model development follows systematic hypothesis testing at each time scale, and generally should lead to
sounder model conceptualisations, with reasonable predictive performance. The hypotheses about model
conceptualisations and emergent properties are tested by focusing on certain signature plots, which are
signatures or patterns of streamflow variability at each time scale, by comparing the observed and model
produced signature plots. Also, it is always attempted to achieve the minimum set of model parameters at each
modelling time scale, which helps to avoid over-parameterisation.

All of the models presented in this chapter are developed based on emergent properties learned or identified
from data in the particular humid and alpine climate found in Austria. The basic model structure therefore
reflects highly specific local characteristics through appropriate modules. Particular features that have been
considered along the way include, 1) snow accumulation and snowmelt, 2) alternating periods of freezing and
thawing of soils in winter, and 3) multiple flow components or pathways: surface runoff, interflow and baseflow.
While the models developed are specific to the study catchment, the modelling approach itself is quite general.

Finally, in the downward approach that is followed in this chapter, it is always aimed for physically sound
parameters, which can be estimated a priori from available field data or regionalised from data from other
locations, or approximately estimated from the literature. This chapter does not specifically deal with the actual
method of estimation of these parameter values, and is limited to the selection of an appropriate model structure
for the catchment in Austria. Future work will deal with the estimation of parameter values, and the effects of
input and parameter uncertainty on model predictions.
2.2 Upper Enns Catchment, Austria: Atmospheric Inputs and Catchment Attributes

In this section, not only the general characteristics of the climate and the physical attributes of the study catchment are described, but also the monitoring network and the data that is available to be used in the various models, and briefly, how these are estimated. A discussion of the key hydrological processes occurring in the catchment is also included, in order to provide motivation for the steps taken later to model the catchment response.

2.2.1 Location, Climate and Hydrology

The study catchment is the Upper Enns catchment with the outlet at Liezen, aligned almost linearly from Southwest to Northeast, and located in the Austrian Alpine region (13.30°-15° E and 47°-48° N) north of the main Alpine ridge (Figure 2.1). The catchment drains an area of 2116 km², with elevations ranging between 700 m a.s.l in the valley bottoms and 3000 m a.s.l at the mountain ridges. The Upper Enns catchment is characterised by Alpine climate, depending on mostly Atlantic, but temporarily also Adriatic, circulation patterns.

The hydrological characteristics of this catchment have been analysed by Hebenstreit (2000). The mean annual precipitation is 1205 mm and the mean annual potential evapotranspiration estimated by the Thornthwaite (1948) method is 489 mm (Table 2.1). Mean actual evapotranspiration is around 267 mm and runoff is 938 mm (see also Figure 2.2 and Table 2.1). Mean monthly precipitation (Figure 2.2) shows strong seasonal variations, with the first maximum in July (160 mm) due to summer storms with high intensities, and a second but smaller peak in December (100 mm), interrupted by the minima in February (60 mm) and October (70 mm). Estimates of mean monthly potential evapotranspiration are also presented in Figure 2.2. The potential evapotranspiration varies seasonally, with a maximum of 95 mm in July and a minimum of almost zero in the winter months of November to March, corresponding to air temperatures below 0° C (Figure 2.3).
Mean monthly precipitation $\langle P \rangle$, mean monthly potential evapotranspiration $\langle E_p \rangle$, and mean monthly discharge $\langle Q \rangle$ standardised by mean annual precipitation $\bar{P}$.

Figure 2.2: Seasonal climatic variations.

Mean daily temperature $T$, mean monthly temperature $\langle T \rangle$, probability normal density function $f(T)$ around $\langle T \rangle$, and critical temperature $T_{crit}$.

Figure 2.3: Mean monthly temperature and fluctuations of daily temperature around the mean.

Budyko (1956) attempted to relate annual estimates of actual evapotranspiration to both annual precipitation and annual potential evapotranspiration, on the basis of observations of over a thousand river basins in the former Soviet Union. His concept is based on previous works of Schreiber (1904) in Central Europe and
Ol'dekop (1911) on Russian rivers. Turc (1945) found a very similar relationship on the basis of measurements in African river basins. The resulting so-called Budyko curve (Figure 2.4) expresses the interesting fact that the partitioning of annual precipitation into actual evapotranspiration and total runoff is a unique function of the (climatic) dryness index or $DI$, which is the ratio of annual potential evapotranspiration to annual precipitation. The Budyko curve allows a rational classification of climate from a hydrologic perspective. Catchments with $DI < 1$ are by definition energy limited (humid), whereas catchments with $DI > 1$ are water limited (semi-arid). The catchments in Australia and New Zealand that have been modelled previously using the downward approach (e.g., Atkinson 2001, Jothityangkoon et al. 2001), are classified as being more arid (more water limited). The study catchment of this chapter, the Upper Enns, has a climatic dryness index smaller than unity ($[\bar{E}_p]/[\bar{P}] = 0.4$) (Figure 2-4), hence is strongly humid (energy limited), with a high runoff coefficient of 0.78 (see Table 2-1).

![Budyko Curve Diagram](image)

Mean annual potential evapotranspiration $[\bar{E}_p]$, mean annual actual evapotranspiration $[\bar{E}_a]$ standardised by mean annual precipitation $[\bar{P}]$. Index of dryness $DI = [\bar{E}_p]/[\bar{P}]$.

**Figure 2.4:** Mean annual actual evapotranspiration ration as a function of the index of dryness by various authors. Positioning of the Upper Enns catchment in this framework.

### 2.2.2 Climatic Model Inputs: Precipitation, Temperature

The meteorological monitoring network of the Austrian Hydrological Survey consists of 40 precipitation and 12 temperature stations in the Upper Enns catchment. Time series from the 1972-1993 period are applied in this study. Based on the point observations, the external-drift-kriging procedure (Ahmed and De Marsily 1987; Cressie 1993) is applied to estimate spatial distributions of precipitation and temperature, with topographic elevation being used as the surrogate variable. Results of the kriging procedure are summarised in terms of lumped catchment estimates of both temperature and precipitation at the annual, monthly and daily time scales, to be used in the water balance models described later.
Table 2.1: Annual climatic and hydrological features of the Upper Enns catchment.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Enns catchment: Liezen, 1972-1993</td>
<td></td>
</tr>
<tr>
<td>Mean annual ( \overline{T} ) [°C]</td>
<td>4.2</td>
</tr>
<tr>
<td>Mean annual ( \overline{P} ) [mm/a]</td>
<td>1205</td>
</tr>
<tr>
<td>Mean annual ( \overline{E}_p ) (Thornthwaite Eq.) [mm/a]</td>
<td>489</td>
</tr>
<tr>
<td>Mean annual ( \overline{E}_a ) ( = [\overline{P}] - [\overline{Q}] ) [mm/a]</td>
<td>267</td>
</tr>
<tr>
<td>Mean annual discharge ( \overline{Q} ) [mm/a]</td>
<td>938</td>
</tr>
<tr>
<td>Runoff coefficient [1]</td>
<td>0.78</td>
</tr>
</tbody>
</table>

For the time period 1972-1993 the catchment-scale mean annual temperature is 4.2 °C. The mean monthly temperatures vary from +12 °C in August to -1.5 °C in January. Temperature has a strong influence on the type of precipitation (rain or snow) and thus on whether or not the precipitated water is immediately available for runoff. For eight months of the year periods of negative mean regional temperatures may occur during the month, especially between November and March. Precipitation in these cold periods falls mostly as snow and accumulate to a temporal snow cover. During the short periods with positive temperatures in winter months, and with generally rising temperatures during spring months, the snow cover is gradually depleted as snowmelt. In spite of the high topographic complexity of the study region and the consequent strong variability of local climatic conditions in space and time, estimated values of the potential evapotranspiration, by the Thornthwaite method (Thornthwaite 1948, Thornthwaite and Mather 1955), are assumed to be spatially uniform across the Upper Enns catchment at all time scales.

### 2.2.3 Rain or Snow

For successful modelling of snowmelt-runoff the correct determination of the form of precipitation is most critical. Standard precipitation measurements do not record the type of precipitation (snow or rain), but only the total amount (snow water equivalent in the case of snowfall). Hence a criterion is necessary to determine the form of precipitation, snow or rain.

For simplicity, the air temperature is taken here as the indicator for determining the form of precipitation, even though it has been realised that the transition air temperature \( T_t \) does exhibit temporal variations within the day as well as through the whole winter season, and also a spatial variability which strongly depends on topographic features. Nachtnebel et al. (1993) for the Upper Enns catchment and Braun (1985) for various catchments in Germany found that this transition air temperature varied in the range -2 °C to +4 °C with a mean of +1 °C. From a 100 year data record Lauscher (1982) found the mean transition air temperature for Vienna of 1.7 °C. Concerning the Austrian Alps own analysis of the data at several stations showed that pure solid precipitation can be expected below approximately -2.5 °C and pure rain may occur above +4.5 °C of mean daily temperature. In between these temperatures, solid and liquid precipitation may alternate within a day or may occur jointly. In this study, as a first approximation, the transition air temperature (which becomes a parameter in some of the models) was fixed at +1 °C.
2.2.4 Runoff Measurements

Runoff measurements are needed to test various hypotheses about runoff generation processes and for the validation of the various models developed in this chapter. The streamflow gauging station at Liezen is selected because this section of the Upper Enns basin is not strongly affected by hydropower operation. The variability of runoff over the year in this catchment is governed strongly by the seasonality of precipitation and seasonal variations of temperature (controlling the type of precipitation and the rate of snowmelt). In addition to Liezen, also streamflow data is used collected at Admont, located 19 stream kilometres downstream of Liezen (Figure 2.1), for the estimation of streamflow recession parameters. The estimation procedure is described in a later section.

2.2.5 Soil Water Storage Characteristics

The lowest section of the river passes through a wide valley built up by gravel. As one follows the river course upstream the alluvial soils are replaced by brown soils on bedrock, soils of small depth above the rock stratum and skeletal soils, which consist of an imperfectly weathered mass of rock fragments with little fine granular material. The soil depth decreases gradually with increasing elevation and slope. In the highest elevation zones the storage capacity of the soils may be neglected and in some parts near the ridges bare rock is exposed to the surface.

Soil properties have been mapped in detail for about 15% of the Upper Enns catchment area (Österreichische Bodenkartierung 1980, 1981, 1985, 1986, 1992). Generally mapped parts include the agricultural areas in the valley bottom and the lower parts of the mountain slopes. For the remaining regions the soil map of the Austrian Academy of Sciences (Österreichische Akademie der Wissenschaften 1979) is available. For areas without detailed field observations this map estimates the soil types based on the local geology, topographic features and climate. Estimates of the local soil properties are comparable with typical values found in Scheffer et al. (1992) and Dingman (1994). In areas near the main river course, an aquifer system with deep groundwater exists, representing about 3% of the total catchment area, and contains within it 16 groundwater monitoring stations.

Table 2.2 presents the properties of soils of the Upper Enns basin. The mean profile porosity $\phi$ is an integrated value over the whole profile depth down to bedrock, $D_{wp}$, or to an impermeable soil layer. The permanent wilting point $\theta_{pwp}$ and the field capacity $\theta_{fc}$ are calculated according to the step-wise methodology of Baumer (1989) for all profile layers of each soil type. This methodology utilises available information on fractions of gravel and rocks, sand, loam, clay, humus as well as the calcium content and the pH value. The spatial fraction for each soil type of the total catchment area is denoted by the $pc$-value. Approximately 1% of the study area shows bare rock, which is mainly located at the highest elevations.
Table 2.2: Soil properties of the Upper Enns catchment.

<table>
<thead>
<tr>
<th>Soil taxonomy</th>
<th>Austria taxonomy</th>
<th>( \phi ) [1]</th>
<th>( D_{tp} ) [mm]</th>
<th>( \theta_{pwp} ) [1]</th>
<th>( \theta_f ) [1]</th>
<th>( pc ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lithosol</td>
<td>Lithosol/Si, Lithosol/Ka, Lithosol/Ka+Si</td>
<td>0.45</td>
<td>200</td>
<td>0.02</td>
<td>0.12</td>
<td>4</td>
</tr>
<tr>
<td>Ranker</td>
<td>Ranker/Si (variation: verbraunter Ranker)</td>
<td>0.45</td>
<td>350</td>
<td>0.05</td>
<td>0.15</td>
<td>5</td>
</tr>
<tr>
<td>Rendzina</td>
<td>Rendzina/Ka, Rendzina/Do, Rendzina/Me (variations: Pararendzina, verbraunte Pararendzina)</td>
<td>0.45</td>
<td>600</td>
<td>0.06</td>
<td>0.14</td>
<td>25</td>
</tr>
<tr>
<td>Cambisol</td>
<td>Braunerde/Si, Braunerde/Sch, Braunerde/Sst, Braunerde/Mo, Braunlehm/Me, Braunlehm/Ka, (variations: kalkhaltige and kalkfreie Lockersediment-Braunerde, kalkfreie Fels-Braunerde)</td>
<td>0.45</td>
<td>800</td>
<td>0.08</td>
<td>0.25</td>
<td>22</td>
</tr>
<tr>
<td>Fluvisol</td>
<td>Grauer Auboden, Brauner Auboden (variation: vergleyter, kalkhaltiger Grauer Auboden)</td>
<td>0.46</td>
<td>2000</td>
<td>0.08</td>
<td>0.30</td>
<td>3</td>
</tr>
<tr>
<td>Podsol</td>
<td>Podsol/Si, Semipodsol/Si</td>
<td>0.47</td>
<td>900</td>
<td>0.05</td>
<td>0.25</td>
<td>36</td>
</tr>
<tr>
<td>Gleysol</td>
<td>Gley (variations: kalkhaltiger Gley, entwässerter kalkfreier Gley, Hangeley)</td>
<td>0.49</td>
<td>1100</td>
<td>0.30</td>
<td>0.32</td>
<td>3</td>
</tr>
<tr>
<td>Histosol</td>
<td>An-, Hoch-, Nieder- and Uebergangsmoor (variations: kalkfreies Niedermoor, entwässertes kalkhaltiges Anmoor)</td>
<td>0.80</td>
<td>1600</td>
<td>0.09</td>
<td>0.15</td>
<td>1</td>
</tr>
</tbody>
</table>

Soil porosity \( \phi \), total depth of the soil profile to an impervious layer \( D_{tp} \), permanent wilting point \( \theta_{pwp} \), field capacity \( \theta_f \), and percentage of area covered by respective soil type \( pc \).

2.2.6 Soil Drainage Characteristics

The models described below introduce, in stages, three mechanisms of runoff generation. Quick runoff \( Q_{se} \) is introduced first and is assumed to occur when the entire soil profile is saturated, i.e., when the total soil moisture capacity of the soil profile \( C_{tp} \) is exceeded by the addition of new incoming water (by means of rainfall or melting snow) to the storage that existed previously. The parameters determining the storage capacity have been discussed in the previous section.

A subsurface runoff component, \( Q_{ss} \), is introduced next, and is initially conceptualised as a linear function of storage, with a mean catchment response time \( t_c \) (inverse of the constant of proportionality of the linear model). This model is later generalised to include two subsurface flow components: an interflow component \( Q_{in} \) and a baseflow component \( Q_{bf} \). The rate of interflow \( Q_{in} \) is similarly governed by a characteristic mean delay time \( t_{c,in} \), and the rate of baseflow \( Q_{bf} \) is governed by a longer delay time \( t_{c,bf} \). To completely specify the model, it is therefore essential that \textit{a priori} estimates of the parameters \( t_c \), \( t_{c,in} \) and \( t_{c,bf} \) are obtained. In this case, these parameters are estimated through an inverse procedure, by making use of recession curves extracted from runoff measurements. Single recession curves, covering a large range of discharges, from high flow to low flow situations at once, without any intervening precipitation event, are joined together to form one master recession curve (Figure 2.5, see also Atkinson 2001).
The estimation of discharge parameters can also be based on observations taken from a nearby gauging station, upstream or downstream of the study site. In this case recession curves of the closest discharge monitoring station to Liezen, which is Admont (Figure 2.1) are also included in the estimation procedure. In Table 2.3 the results of the recession analyses at station Admont are presented. For the sake of comparison the respective values referring to gauging station Liezen are shown in brackets. Both linear as well as non-linear recession functions are fitted to the observed master recession curves (Wittenberg 1994). The procedure is similar to that described by Atkinson (2001) for a non-linear storage-discharge model.

### Table 2.3: Catchment response times for total flow \( t_c \), interflow \( t_{c,\text{in}} \) and baseflow \( t_{c,\text{bf}} \) at gauging station Admont (results for Liezen in brackets for comparison).

<table>
<thead>
<tr>
<th>Catchment response times</th>
<th>[days]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total runoff ( t_c )</td>
<td>21(20)</td>
</tr>
<tr>
<td>Interflow ( t_{c,\text{in}} )</td>
<td>13(12)</td>
</tr>
<tr>
<td>Baseflow ( t_{c,\text{bf}} )</td>
<td>46(39)</td>
</tr>
</tbody>
</table>

#### 2.2.7 Parameters Affecting Snowmelt and Accumulation

In this study, as a first approximation, a simple model is utilised to account for the processes determining snow accumulation and depletion that uses the air temperature as a surrogate variable. The model conceptualisation is based on two parameters: the degree-day factor or melt factor \( mf \) and a threshold air temperature \( T_o \). Braun (1985) investigated optimal values for both parameters in four river basins, Rietholzbach (Riet), Thrur (Adelfingen), Murg (Wängi) and Thur (Jonschwil), for accumulation as well as melt periods. He reports a range
of optimal threshold air temperatures between 0 °C and 1.8 °C, and melt factors in the range of 2.0 to 4.5 mm/K/d. Fuchs (1998) found similar values by developing and applying different types of snow melt models to the Enns catchment.

The threshold air temperatures are similar in magnitude to the transition air temperatures used in a previous section to determine the form of precipitation as either snow or rain. For the sake of model parsimony and based on the suggestion of Bergström (1976), the threshold and transition air temperatures \( T_o \) and \( T_r \) are collapsed into a single critical temperature \( T_{crit} \). The chosen parameter values are presented in Table 2.4, and are not modified at all with changing model structures at the various time scales.

Table 2.4: Snow model parameters.

<table>
<thead>
<tr>
<th>Snow parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Melt factor ( mf ) [mm/K/d]</td>
</tr>
<tr>
<td>Critical temperature ( T_{crit} ) [°C]</td>
</tr>
</tbody>
</table>

2.3 Emergent Properties, Signature Plots and the Downward Approach

Model complexity required to make predictions of runoff variability at the annual, monthly and daily time scales is investigated. In this section the principles that have been followed are described and discussed:

2.3.1 Climatic Inputs and Model Time Step

In all of the models presented in this chapter, the water balance computations and flux estimations are carried out using a daily time step. However, the models are classified as annual, monthly or daily on the basis of the input data used. The annual model, for example, will only take in annual data as input, such as measured or estimated rainfall, potential evapotranspiration, and any other climatic statistics that describes within-year variability of all the relevant quantities, such as the average number of rainy days etc. In other words, actual daily or monthly time series of rainfall are not needed as input to run the annual model. Similarly, the monthly model takes in monthly inputs of observed and estimated rainfall and potential evapotranspiration, and any statistics describing within-month variability of all relevant climatic variables, such as the daily or average monthly temperature, the probability distribution of temperatures and the mean number of rainy days in each month etc. The daily model, of course, will use observed and estimated daily time series of all the relevant quantities. In principle it could use mean climatic information about within-daily information such as the duration of sunshine hours or rainfall, but these have not been used in this study.

2.3.2 Catchment Attributes

Regardless of the time scale of the model (annual, monthly and daily), parameters of the model are seen as physical attributes of the catchment that are measured or are measurable, and thus are not allowed to change with time scale. Therefore, once a parameter value has been defined at one time scale, it will not be changed at any other time scale. On the other hand, a parameter or attribute that was not utilised at one time scale may become important at a smaller time scale.
2.3.3 Parameter Estimation

An important requirement is that the parameters are estimated prior to the application of any of the models, and not through optimisation of the fits between model predictions and observations. Available information to estimate the required parameters is utilised at all times, and in the case of the drainage properties, these are estimated from observed streamflow recession curves.

2.3.4 Emergent Properties

The concept of emergent properties in the context of catchment hydrology is described next. Here, emergent properties is defined as those processes and associated process controls (catchment attributes) that one encounters when the scales of observation are changes, be it time scales or space scales. This chapter limits itself to time scale changes. Emergent properties cause problems for modelling when accepted physical theories are developed at one scale, and these are no longer adequate at a higher or lower level of scale. To begin with, a simple overflow bucket model (Manabe 1969) is usually deemed sufficient to adequately predict variability of runoff at the annual time scale, such as mean annual runoff and the inter-annual variability of annual runoff. The inter-annual variability plot is the key signature for model testing. For example, in humid regions the Manabe bucket is a reasonable model, as shown by Milly (1994). At the monthly time scale, it could be found conceivably that it is important to capture delayed flow, as otherwise it would not be able to capture the monthly distribution of runoff. The observed delays can be caused by flows through subsurface pathways, and these could also be caused by snow accumulation and subsequent snowmelt because of the peculiar characteristics of boreal catchments. Hence, in this instance, subsurface flow and snow processes are both considered as emergent properties. In this case, one could try to associate these emergent properties, and find ways to identify them, with the “signature plot” at the monthly scale.

However, at the daily time scale, flow peaks and the falling limbs of the hydrographs have to be modelled and this requires the introduction of multiple pathways for the flow through soils: a baseflow component which occurs at all times so long as there is water remaining in the hillslopes, and an interflow component which occurs when soil water storage exceeds the threshold storage corresponding to field capacity. The need for explicit treatment of soil freezing and thawing was also investigated, but found to be insignificant in the study catchment and deemed unnecessary for lumped modelling. These are the possible new emergent properties at the daily time scale, and the ability of the model to incorporate these emergent properties needs to be tested using a new signature plot that is able to capture the ability of the model to describe both peak flows as well as low flows. An example of such a signature plot is the so-called flow duration curve, estimated on daily flows.

2.3.5 Methodology of the Downward Approach

Having defined the likely evolution of emergent properties at the annual, monthly and daily time scales, and the signature plots that will be used along the way, in sequence, to test the ability of the evolutionary model to match observed runoff variability, it is now easier to define or specify the form of the downward approach to model development. In the context of the above, the downward approach involves starting with a simple model at the annual time scale, and as one goes down to the monthly time scale, the modeller goes through a sequence of steps of postulating the likely emergent properties that may address the problem of poor fits when using the
previous model in the evolution. Any modification of the model does not mean the model structure used previously is jettisoned – rather it is built on what was previously used, and carefully and with adequate justification and testing new processes to match the new signature plots are added. At every time scale, it is made sure that the model is parsimonious – no more complex than is necessary. If the model matched data adequately, then adding another process will lead to over-parameterisation and increased predictive uncertainty. The process continues on down to the daily or even to the hourly time scales.

2.4 Modelling Framework and Key Assumptions

2.4.1 Soil Moisture Bucket

The models presented in this chapter are all based on the “bucket” concept (Nash 1957; Sugawara 1995). The most fundamental building block, or the simplest model, is the Manabe bucket model (Manabe 1969). For example, at the annual time scale, the Manabe bucket model has already been shown to be an adequate model for the humid or energy limited catchments in North America (Milly 1994). In semi-arid or water-limited climates, the Manabe bucket may no longer be adequate, even at annual time scales, as shown in a previous work in Australia (Jothityangkoon et al. 2001).

As the time scale decreases to the monthly and daily, model complexity will evolve from the Manabe bucket model, invariably becoming more complex. There are a number of ways that model complexity can be increased. These can be listed at the outset, although not all these aspects will be included in the final model presented.

− introduction of subsurface flow, in addition to saturation excess runoff,
− combination of buckets of different sizes so as to mimic partial area runoff generation,
− introduction of multiple subsurface flow pathways, such as shallow subsurface flow and deeper baseflow,
− introduction of delays in the unsaturated zone,
− separation of total evapotranspiration into bare soil evaporation, transpiration and interception loss,
− introduction of channel flow (streamflow routing), and
− introduction of snow accumulation and snowmelt, and soil freezing and thawing.

2.4.2 Climatic Forcing

The water balance models presented here are based on two or three climatic input data sets only, namely, precipitation $P$ and potential evapotranspiration $E_p$, and air temperature $T$. For the daily model, observed daily time series of each of these quantities are used. For the annual and monthly models, only annual and monthly inputs are used, respectively, along with rather simple statistics describing, respectively, within-year and within-month variations of these quantities. For example, in the annual water balance model, the year is divided into a wet season and a dry season and all of the annual precipitation is assumed to fall uniformly in the wet season, and all of the potential evapotranspiration applies over the dry period, again uniformly in time and space.
2.4.3 Evapotranspiration in Energy Limited Catchments

All the models presented in this chapter assume that evapotranspiration is negligible during rainfall periods, since 1) the atmosphere is near its saturated state, and due to extensive cloud cover, solar energy is also very limited, and potential evapotranspiration rates are very small, and 2) the rainfall rate (during rainy periods) far exceeds any evapotranspiration that may actually happen. In addition to the above, during non-rainy or dry periods, actual evapotranspiration is assumed to be equal to the potential evapotranspiration. This is a simplification adopted for the Upper Enns catchment, and is a consequence of the fact that throughout the year the catchment soil moisture is maintained at near saturation levels, and there is little soil moisture control on the rate of evapotranspiration rate. This makes the model very simple and parsimonious.

2.5 Annual Water Balance

Milly (1994) showed that over 80% annual water balance variability in the eastern half of the United States can be captured by the simple Manabe bucket model. Consequently, this is the starting point for the annual model development in this chapter. Since empirical evidence suggests that the soils are saturated or nearly so throughout the year, the number of days of no rain (or still more precisely, total number of hours of sunshine) would be the most dominant factor governing annual evapotranspiration, and hence annual water balance.

2.5.1 Development of Annual Water Balance Model

In the annual model the atmospheric fluxes are conceptualised in terms of alternating wet-dry periods. Wet periods receive precipitation at a constant rate, as shown in Figure 2.6a and 2.6b, but do not evaporate, and dry periods do not receive precipitation but evaporate at the potential rate, assumed constant. In this way, the climatic forcing is based on two key variables, namely the lengths of the wet and dry periods (Figure 2.6c and 2.6d), which can be estimated with confidence from a few years’ recordings or estimates of precipitation and potential evapotranspiration (Table 2.5).
The Manabe bucket model ignores infiltration excess runoff and subsurface flow, and only includes a form of saturation excess runoff. Following Manabe (1969), the storage capacity of the soil is represented by a single bucket, which can store a limited amount of water (Figure 2.7). Saturation excess runoff $Q_{se}$ is produced when the soil moisture storage at any time becomes higher than the total soil profile capacity.

In the Austrian Alpine region precipitation occurs in the form of frequent short events throughout the whole year, but the frequency and intensity of events also show seasonal variations. Therefore two diametrically opposite approaches are tested to capture, in a simple manner, this within-year variability of climatic inputs, and to test if the frequency of the precipitation forcing is a determining factor for the annual water balance: the concept of *seasonality*, which refers to just two seasons of wet and dry periods in the whole of year, and the concept of *storminess*, using a larger number of alternating wet and dry periods, each representing individual storms. In the former case, an extreme version of seasonality is included, but the intermittence of storms is disregarded. In the latter case, intermittence of storms is included, but the seasonality is ignored. In the concept of *seasonality* the length of the wet season is equal to the number of days per year with precipitation ($t_{wet}$). The total annual precipitation is then spread uniformly over the wet season only, followed by the dry season without

![Figure 2.6: Concept of precipitation forcing for the annual (a) and monthly water balance (b): Alternating process of wet and dry periods (a, c) or storm and inter-storm periods (b, d).](image-url)
precipitation for the remaining part of the year \((t_{\text{dry}})\) (Figure 2.6). In the concept of \textit{storminess} precipitation falls during storm events of several successive days \((t_s)\), each of which is followed by an inter-storm period \((t_{\text{ist}})\).

\[
P \quad E_a \quad Q_{\text{se}}
\]

Precipitation \(P\), actual evapotranspiration \(E_a\), saturation excess runoff \(Q_{\text{se}}\), total soil moisture capacity of the soil profile above the permanent wilting point \(C_{wp}\), and soil moisture storage level \(S\).

**Figure 2.7:** The Manabe type bucket accounting for soil moisture storage, actual evapotranspiration and saturation excess runoff.

**Table 2.5:** Long term mean annual precipitation characteristics for catchment climate forcing.

<table>
<thead>
<tr>
<th>Mean annual values</th>
<th>[days]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of days with precipitation ([\bar{t}_{\text{wet}}])</td>
<td>198</td>
</tr>
<tr>
<td>Number of days without precipitation ([\bar{t}_{\text{dry}}])</td>
<td>167</td>
</tr>
<tr>
<td>Length of storm event ([\bar{t}_s])</td>
<td>3</td>
</tr>
<tr>
<td>Length of inter-storm period ([\bar{t}_{\text{ist}}])</td>
<td>4</td>
</tr>
</tbody>
</table>

### 2.5.2 Mathematical Formulation of Annual Model

Two different model formulations are used, depending on the climatic forcing used. In the case of the \textit{storminess} concept, rainfall in the year is assumed to consist of a number of identical storms, and the long-term means of individual storm durations \([\bar{t}_s]\) and inter-storm periods \([\bar{t}_{\text{ist}}]\) are used as storm properties. The number of storms in the year, defined as \([m]\), is approximately \([m] = J / [\bar{t}_s + \bar{t}_{\text{ist}}]\), where \(J\) is the length of the year. In the case of the \textit{seasonality} concept, there is effectively only one storm with a duration equal to the long-term mean wet season length \([\bar{t}_{\text{wet}}]\), given by \([\bar{m}],[\bar{t}_s]\), while the inter-storm period is equal to the length of the dry season, which does not need to be separately parameterised since it is the difference between the length of the year and the wet season length \([\bar{t}_{\text{dry}}]\), i.e., is equal to \(J - [\bar{m}],[\bar{t}_s]\). The soil depth \(D_p\), the profile porosity \(\phi\) and the permanent wilting point \(\theta_{\text{pwp}}\) of all representative soil types identified within the study catchment are used next to estimate the mean bucket capacity for use in the model.

The model formulation will be presented in a general manner, applicable to both the \textit{storminess} and \textit{seasonality} concepts presented before. It begins with the presentation of the water balance equation for the lumped bucket model, using a daily time step \((\Delta t = 1)\). The annual water balance concept effectively has only five climatic and soil parameters: \([m]\), \([\bar{t}_s]\), \(C_{wp}\) (estimated from \(D_p\), \(\phi\) and \(\theta_{\text{pwp}}\)).
Water Balance Equation

\[ S(t+1) = S(t) + P(t) \Delta t - Q_a(t) \Delta t - E_a(t) \Delta t \]  \hspace{1cm} (2.1)

**Precipitation**

Regardless of the climatic forcing used, the daily precipitation intensity to be used in the above equation, during wet or rainy periods is given by

\[ P = \frac{[P]}{[m]} \]  \hspace{1cm} (2.2)

**Actual Evapotranspiration**

\[ E_a = \text{Min}\{E_p; S/\Delta t\} \quad \text{when } P = 0 \]
\[ E_a = 0 \quad \text{when } P \neq 0 \]  \hspace{1cm} (2.3)

where \( E_p \) is the daily potential evapotranspiration rate, as estimated below and assumed to be uniform throughout the year.

**Potential Evapotranspiration**

\[ E_p = \frac{[E_p]}{J} \]  \hspace{1cm} (2.4)

\[ J \ldots \text{number of days per year} \]

**Saturation Excess Runoff**

Saturation excess runoff is produced if and when the net additions to the bucket via precipitation and evaporation are such that the storage of water in the bucket exceeds the capacity of the bucket, denoted by \( C_p \). Thus, the rate of runoff generation is given by:

\[ Q_{se} = \text{Max}\{0; (S - C_p)/\Delta t\} \]  \hspace{1cm} (2.5)

where \( C_p \) is estimated as follows:

**Bucket Storage Capacity**

\[ C_p = \sum_{i=1}^I D_{ip} (\phi_i - \theta_{pwp}) \cdot p_c_i \]  \hspace{1cm} (2.6)

\[ I \ldots \text{number of different soil types} \]

Note that the model is operated on a daily time step, but using only annual climatic inputs of precipitation and potential evapotranspiration, and mean annual climatic parameters such as the mean storm duration and mean inter-storm period and the three soil parameters: soil depth \( D_{ip} \), porosity \( \phi \) and permanent wilting point \( \theta_{pwp} \).

### 2.5.3 Discussion of Annual Model Results

The simulations showed, firstly, that there was no significant difference between the model based on *seasonality* and that based on *storminess*. The main reason for this that for the size of the bucket capacity used, the moisture storage in the bucket never completely empties in either version of the annual model, although in general the chances for emptying are more in the case of the seasonality concept because of the longer sustained dry period. However, this did not happen in this catchment. On the other hand, in the *storminess* formulation, the soil water storage dries out to a lesser extent, and thus over the year the soil water storage always tends to fluctuate at or just below the maximum capacity. A consequence of the fact that the bucket never empties is that the total annual evapotranspiration, according to the above simple model, is equal to rate of potential evapotranspiration.
multiplied by the length of the dry period. This means that actual evapotranspiration is the same in both model concepts, since the total length of dry period is identical in both cases.

![Graph](image)

(a) Inter-annual variability of annual yield
(b) Annual streamflow hydrograph

Annual discharge $[Q]$ (observed $Q_o$ and modelled discharge $Q_p$) standardised by mean annual precipitation $[\overline{P}]$.

**Figure 2.8: Performance of the annual water balance model considering saturation excess runoff only.**

The results of the annual water balance model are presented in Figure 2.8 (only the seasonal concept is presented, for the sake of brevity). In spite of the simplicity of the model (using annual precipitation time series and summary statistics of mean number of rainy days) and a single bucket capacity, the model captures the general trend of the statistical, between-year variability, as seen in the inter-annual variability plot (Figure 2.8a). However, the comparison between observed and modelled annual time series plots shows that individual years are not all reproduced well (Figure 2.8b). These suggest that within-year processes may be more complex, and may also be variable between years, and the simple representations of within-year variability presented earlier in terms of the concepts of *storminess* and *seasonality* are not entirely satisfactory.

### 2.6 Seasonal Water Balance

A new feature that enters the problem when modelling at the monthly time scale is the within-year variation of both precipitation and potential evapotranspiration (and temperature). However this alone should not make much of a difference to the annual water balance, as shown for the model previously was worked on. An additional feature that enters the hydrology in the Upper Enns catchment, being located in a cold region, is the fact that during parts of the year the precipitation could fall in the form of snow, so long as temperatures remain below freezing, leading to their temporary accumulation on the ground as snowpack. Once the temperatures warm up during the spring, the snowpack begins to melt, and contributes to both soil moisture storage and snowmelt runoff. Snow accumulation in winter and melt in spring introduce elements of carry-over of storage and delays to the system.

The combination of snow accumulation and melt is therefore the first mechanism to explore, but initially using the same bucket overflow model as presented before. The next obvious candidate to consider (in case the above extension is not sufficient to capture annual and monthly runoff) is the possibility of additional delays in
the system due to subsurface flow pathways, travel times in the river system etc. This is the second mechanism that will be investigated in the model development.

2.6.1 Development of Seasonal Model

In place of annual average values for the lengths of storm and inter-storm periods, values that vary monthly are adopted. The storm duration varies seasonally with the longest rain periods in July and the shortest in September and October. The variations of the total number of wet and dry days per month show a similar trend. The lengths of storm and inter-storm periods as well as the lengths of the wet and dry seasons are obtained by analysis of the regional precipitation characteristics (Table 2.6). Both the concepts of storminess and seasonality are now slightly adapted in the monthly model. Precipitation is assumed to fall in wet-dry cycles, either in one single wet period per month or in a number of identical storm events. These concepts incorporate a considerable degree of realism regarding precipitation variability throughout a year, and result in similar model performances. Here, for illustration, only results from the monthly concept of storminess are presented.

Table 2.6: Long term mean monthly precipitation characteristics for catchment climate forcing.

<table>
<thead>
<tr>
<th>Mean monthly values [days]</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of wet days $\langle t_{\text{wet}} \rangle$</td>
<td>13</td>
<td>11</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>19</td>
<td>19</td>
<td>16</td>
<td>13</td>
<td>10</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>Number of dry days $\langle t_{\text{dry}} \rangle$</td>
<td>18</td>
<td>17</td>
<td>18</td>
<td>16</td>
<td>16</td>
<td>11</td>
<td>12</td>
<td>15</td>
<td>17</td>
<td>21</td>
<td>18</td>
<td>17</td>
</tr>
<tr>
<td>Storm length $\langle t_{s} \rangle$</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Inter-storm length $\langle t_{s-s} \rangle$</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

First the annual model described before (overflow bucket) using the monthly inputs is operated, as described here. In this case, simulations of mean monthly runoff (not presented) did not match the observations. Observed flows were dominated by high flows from spring to early summer (presumably due to snowmelt) and by low but sustained flows in winter when precipitation occurs mostly in the form of snow, and due to the low temperatures, accumulates to a snow pack. The annual model when applied with monthly data over-predicted the flows in winter and under-predicted in spring and early summer, and peak flows are delayed by about 2 months as well. In other words, the model did not capture the observed delays between precipitation and runoff.

Next a simple extension of the model is introduced to partition precipitation into rain and snow (using a critical temperature $T_{\text{crit}}$ of 1.0 °C), and a snowmelt algorithm based on the melt factor, $mf$, of 1.5 mm/K/day (World Meteorological Organization 1986). The temperature was assumed to be a constant throughout the month and equal to the observed monthly average temperature. Runoff generation continues to be through overflow of the bucket, leading to saturation excess runoff only. Details of this model or its detailed predictions are not presented since this is a subset of the extended monthly model presented later in this section. The predictions with this model were significantly better than the previous model. Now the timing of the seasonal variations improved yet did not exactly match the observations. Moreover, both the low flows and the high flows were more extreme compared to the observations: high flows were over-predicted and low flows were under-predicted. This confirms, 1) use of average monthly temperature seems to have over-predicted the amount snow
accumulation, and 2) a delayed flow component is still missing especially in summer flows, pointing to subsurface flow pathways.

In the case of water balance modelling using a monthly time step, the estimation of snow accumulation and depletion processes on the basis of monthly average temperatures did not allow for short-term accumulation and melt processes, during short periods with temperatures below and above \( T_{\text{crit}} \), respectively. Analysis of the temperature distributions within individual months showed that for all months mean daily temperatures are normally distributed, the standard deviation, \( \sigma_T \), of which varies seasonally around 4 °C (see Figure 2.3).

Therefore, the model described previously is extended by relaxing the assumption of constant temperature within the month, and assuming normally distributed temperatures within the month. Thus, even if the monthly average temperature is far below the critical temperature there will still be short-term melting periods within the month, which can give rise to snowmelt, and *vice versa* for snow accumulation. In this modified monthly model, the assumed critical temperature splits the distribution function into two parts: above and below the critical temperature, \( T_{\text{crit}} \). The volumes of snow accumulation and depletion within the month are assumed to be proportional to the respective integrals of the probability density function of temperatures over the two regions defined as above, i.e., below \( T_{\text{crit}} \) (for snow accumulation) and above \( T_{\text{crit}} \) (for snow depletion). In effect they measure cumulative degree days below and above the threshold temperature, respectively. Key details of the model are described only in the next section, for the sake of brevity, while here a sequence of modelling ideas and results are presented. With the modified model, the timing of the mean seasonal variations of runoff (i.e., regime curve) almost exactly matched the observations. This showed that the seasonal snow accumulation and snowmelt processes are now simulated better than previously. Next, a comparison of actual monthly flows predicted by this latest version of the model against observed flows was carried out. These results showed, however, that while low flows are generally under-predicted every year in winter, high flows are over-predicted in late spring and early summer. The conclusion from this is that a delayed flow component was still missing, pointing to the inclusion of sub-surface flow pathways.

**2.6.2. Mathematical Formulation of Monthly Model**

In the spirit of the downward approach, the preliminary conclusions drawn in the last section led to the postulation of the revised model structure presented in Figure 2.9. In addition to the existing saturation excess runoff component, \( Q_{se} \), a sub-surface flow component \( Q_{ss} \) is introduced. The model also has the snow accumulation component discussed above, including the use of normally distributed temperatures within the month and the use of the critical temperature \( T_{\text{crit}} \) and the melt factor for snowmelt, \( mf \). The introduction of subsurface runoff necessitated the estimation of a time delay or response time of the catchment. This response time is connected overall to the drainage properties of the catchment, such as the hydraulic conductivity of the soil, steepness of topography, convergence features, and above all the presence of macropores and other preferred pathways and heterogeneities. Since at this stage these are hard to measure and estimate, the residence time from recession analysis carried out on observed streamflows is estimated, as previously described in section 2.6.
The monthly model is based on nine model parameters: \(\bar{m}\), \(\bar{T}_r\), \(\sigma_T\), \(C_{ip}\) (\(D_{ip}\), \(\phi\) and \(\theta_{pwp}\)), \(T_{crit}\), \(mf\) and \(t_c\).

\[
T \leq T_{crit} \quad P_s \\
T > T_{crit} \quad P_r, E_r
\]

Temperature \(T\), critical temperature \(T_{crit}\), precipitation falling as snow \(P_s\) or as rain \(P_r\), snowmelt runoff at thawing conditions \(Q_N\), snow water equivalent in the snowpack \(S_N\), actual evapotranspiration \(E_a\), total soil moisture capacity of the soil profile above the permanent wilting point \(C_{ip}\), soil moisture storage level \(S\), and saturation excess runoff \(Q_{se}\), catchment response time \(t_c\) of sub-surface runoff \(Q_{ss}\).

**Figure 2.9:** The monthly concept accounting for snow processes, saturation excess runoff and a sub-surface runoff component.

**Partitioning of Precipitation into Snow and Rain**

In order to do this, the probability density function of air temperatures for every month is used to obtain cumulative degree-days above and below the specified critical temperature, \(T_{crit}\) (accumulated positive temperature days and negative temperature days). These are defined and estimated as follows:

\[
\langle H_{pos}\rangle = \sum_{j=1}^{c} n_j (T_j - T_{crit}) \quad \forall T_j > T_{crit} \quad (2.7)
\]

\[
\langle H_{neg}\rangle = \sum_{j=1}^{c} n_j (T_{crit} - T_j) \quad \forall T_j \leq T_{crit} \quad (2.8)
\]

- \(c\) ...number of temperature classes in the temperature histogram of mean daily temperatures in the month [1].
- \(j\) ...a temperature class in the histogram.
- \(n_j\) ...number of days belonging to class \(c_j\) [1].

The total monthly precipitation depth is then partitioned into snowfall \(\langle P_s \rangle\) and rainfall \(\langle P_r \rangle\) according to the following equation, with the accumulated positive and negative temperature degree-days being assumed to represent the relative fractions of rainfall and snowfall within the month.

\[
\langle P_s \rangle = \langle P \rangle \left( \frac{\langle H_{neg} \rangle}{\langle H_{pos} \rangle + \langle H_{neg} \rangle} \right) \quad (2.9)
\]
In the monthly model both rainfall and snowfall are assumed to be constant within the wet periods; in other words, provided there is both snow and rain in the month, they are assumed to occur uniformly (at a constant rate) and concurrently. Thus the intensity of daily snowfall and rainfall assumed are given by:

\[ P_s = \frac{\langle P_s \rangle}{m} \bar{t}_w \]  
\[ P_r = \frac{\langle P_r \rangle}{m} \bar{t}_w \]  

**Daily Potential Evapotranspiration Rate**

\[ E_p = \frac{\langle E_p \rangle}{N} \]  

where \( N \) is the number of days in the month

**Water Balance of Soil Moisture Store and Snow Store**

The water balance of the soil moisture store is updated daily during both wet and dry periods using the following equation:

\[ S(t+1) = S(t) + P(t)\Delta t - E_a(t)\Delta t - Q_{se}(t)\Delta t - Q_{ss}(t)\Delta t + Q_N(t)\Delta t \]  

Similarly, the snow water equivalent in the snowpack is updated daily using the following water balance equation:

\[ S_N(t+1) = S_N(t) + P_s(t)\Delta t - Q_N(t)\Delta t \]  

In the above two equations, surface runoff, \( Q_{se} \), is estimated by using Equation 2.5, as before in the annual model, and is not repeated here. Similarly, actual evapotranspiration \( E_a \) is estimated using Equations 2.3 and 2.4. The remaining terms \( Q_N \) and \( Q_{ss} \) are estimated as given below. Note also that \( Q_N \) appears in both water balance equations as the melted water is assumed to contribute to the soil water store rather than flow directly to the river.

**Rate of Snowmelt**

\[ Q_N = \text{Min} \left\{ \frac{\langle H_{pos} \rangle}{N}; S_N / \Delta t \right\} \]  

Clearly, this is a crude model as far as the snow processes concerned, assuming that both rain and snow fall uniformly during precipitation periods, and that snowmelt occurs throughout the month during both wet and dry periods, also at a uniform rate provided there is snow on the ground.

**Subsurface Flow**

\[ Q_{ss} = \frac{S}{t_c} \]  

**2.6.3. Discussion of Monthly Model Results**

The performance of the final version of the monthly water balance model is investigated through comparison of both signature plots and runoff time series at the annual and monthly time scales. Both the inter-annual
variability plot (Figure 2.10a) and the annual streamflow hydrograph (Figure 2.10b) are now significantly better than the corresponding results for the early version models (not presented). However, there are still considerable differences between the observations and runoff predicted by the monthly model, which only uses mean monthly statistics for the number of storms or the lengths of the wet and dry periods. This reasoning suggests that the model can improve by going down to a daily time step and with the use of actual (rather than assumed uniform) climatic inputs. Finally, with the introduction of subsurface flow the monthly streamflow hydrographs are significantly better (Figure 2.10d).

Figure 2.10: Performance of the monthly water balance concept considering saturation excess runoff and a delayed sub-surface flow component.

However, even with this enhanced model (snow processes and subsurface flow), the mean monthly predictions did not exactly match the observations. Indeed, there has been a deterioration compared to the early version models. In particular, there appears to be slight under-estimation in the winter months and early spring (January, February, March, April, May and June), and slight over-prediction in the summer (July, August, September and October). Two potential causes of these discrepancies were investigated. The first is the likelihood of soil freezing over in winter and preventing infiltration of water, and the thawing of frozen soil in spring. This was investigated through a simple model, which produced only marginal improvements to the model.
predictions. Indeed, field experience in this region suggests that freezing of soil in winter is rare except in elevations above 1800 m a.s.l. For these reasons, the possibility of the freezing and thawing of soils was discounted. The second explanation is that perhaps the assumed subsurface residence time was too long, and that improved results may be obtained by breaking up the subsurface component into a fast interflow component and a slow baseflow component. This is considered in the daily version of the model presented next.

2.7. Daily Water Balance

At the daily time scale runoff time series exhibit much more variability in terms of high flows and low flows, and the transition between them. The extremes are connected to the random nature of storm events, which does not appear at the annual or monthly time scales, combined with the effect of antecedent soil moisture conditions, and the feedback between runoff processes and soil moisture storage. Hence, model complexity must increase at the daily time scale, especially requiring better definition of runoff processes (pathways) occurring at a range of time scales, including possibly the explicit treatment of time delays in the stream network (streamflow routing). Also, the relative components of overland flow, subsurface flow, baseflow and channel flow may change with changes of soil moisture storage, and with size of catchment.

2.7.1. Development of Daily Model

As a starting point, the latest version of the monthly model at the daily time step was tested, now using observed precipitation and potential evaporation time series. The results presented in Figure 2.11 indicate that while the mean flows were slightly over-predicted, the low flows and the high flows (peaks) were under-predicted by the model, demonstrated best by the flow duration curve. This confirms the previous suspicions that the single subsurface residence time $t_c$ may have been too large and may tend to dampen the high flows.
2.7.2. Mathematical Formulation of Daily Model

The new bucket model is built around eight parameters only: $C_{fc}$ ($D_{wp}$, $\phi$ and $\theta_{fc}$), $C_{wp}$ ($D_{wp}$, $\phi$ and $\theta_{wp}$), $T_{crit}$, $mf$, $t_{c,in}$ and $t_{c,bf}$. Only the new additions to the model are described below. In all other respects the model is identical to what was presented before for the monthly model.
Temperature $T$, critical temperature $T_{crit}$, precipitation falling as snow $P_s$ or as rain $P_r$, snowmelt runoff at thawing conditions $Q_N$, snow water equivalent in the snowpack $S_N$, actual evapotranspiration $E_a$, total soil moisture capacity of the soil profile above the permanent wilting point $C_{wp}$, soil moisture capacity until field capacity $C_{fc}$, soil moisture storage level $S$, saturation excess runoff $Q_{se}$, catchment response time $t_{c,se}$ of interflow $Q_{in}$, and catchment response time $t_{c,bf}$ of the baseflow component $Q_{bf}$.

Figure 2.12: Model concept accounting for snow processes, saturation excess runoff, interflow and a base flow component.

Water Balance of Soil Moisture Store in the Daily Model

$$S(t + 1) = S(t) + P_s(t)\Delta t - E_a(t)\Delta t - Q_{se}(t)\Delta t - Q_{in}(t)\Delta t + Q_{bf}(t)\Delta t$$  \hspace{1cm} (2.18)

Form of Precipitation

$$P_r = P \quad \text{if } T > T_{crit}$$
$$P_s = P \quad \text{if } T \leq T_{crit}$$  \hspace{1cm} (2.19)

Interflow

$$Q_{in} = \frac{S - C_{fc}}{t_{c,in}}$$  \hspace{1cm} (2.20)

Baseflow

$$Q_{bf} = \frac{S}{t_{c,bf}}$$  \hspace{1cm} (2.21)

Rate of Snowmelt

$$Q_N = \text{Min}\{H_{pos} \cdot mf; S_N / \Delta t\}$$  \hspace{1cm} (2.22)

where $H_{pos}$ is a measure of the energy driving the snowmelt to be used in combination with the melt factor $mf$, and is now variable on a daily basis. It is estimated as the temperature excess over the critical temperature, $T_{crit}$, as given below.

Degree-Day Magnitude above Critical Temperature

$$H_{pos} = \text{Max}\{T - T_{crit}, 0\}$$  \hspace{1cm} (2.23)
2.7.3. Discussion of Daily Model Results

The results of the daily model application are presented in Figure 2.13a-f. It is clear that a parsimonious model with just six parameters has managed to give good reproductions of the daily, monthly and annual hydrographs as well as the three signature plots. There has been an improvement in the daily model predictions, including the flow duration curve. In particular, it has been shown that the flow duration curve could not be matched if only one subsurface flow component with a single residence time of $t_c$ was considered. The introduction of the two flow components of interflow and baseflow has produced improved predictions. For completeness, the continuous fluctuations of the snow water equivalent and the soil moisture storage are presented in Figure 2.14b, in relation to the two determining soil parameters, the total profile water holding capacity and the field capacity. These results confirm previous assertions about the general soil moisture dynamics and the variability of snow accumulation and depletion in this catchment. Figure 2.14a shows the simulated runoff components baseflow and interflow in comparison to snowmelt over a two-year period.

Finally, it should be reported that attempts at adding further complexity to the daily water balance model presented above did not lead to substantial improvements in model performance. It was felt that inclusion of more complexity at this stage can only lead to more uncertainty in the simulations because of the need to estimate additional parameters. This led to the conclusion that any more improvements can only come from accounting for spatial variations of physical and meteorological features of the catchment. This was left for future work.
Annual $[Q]$, mean monthly $\langle Q \rangle$, monthly $\langle Q \rangle$, and daily discharge $Q$ (observed $Q_o$ and modelled discharge $Q_p$) standardised by mean annual precipitation $[P]$.

Figure 2.13: Performance of the daily water balance concept considering saturation excess runoff inter flow and a base flow component.
2 Water Balance Modeling Through Emergent Properties

Daily snowmelt runoff $Q_N$, interflow $Q_{in}$, baseflow $Q_{bf}$, snow water equivalent in the snowpack $S_N$, soil moisture storage level $S$, total soil moisture capacity of the soil profile above the permanent wilting point $C_{wp}$, soil moisture capacity until field capacity $C_{fc}$ standardised by mean annual precipitation $\bar{P}$.

Figure 2.14: Modelled runoff components interflow, baseflow and snow melt (a). Fluctuation of the state variables snow water equivalent in the snowpack and soil water storage around the field capacity and the total capacity of the soil profile (b).

2.8. Summary and Conclusions

This chapter has presented a methodology for developing predictive water balance models based on analysis of observations. The aim was to develop parsimonious water balance models, which use a minimum number of input variables and model parameters. This goal was reached by following a downward approach, starting from the annual time scale and gradually evolving to the monthly and daily time scales, updating the model structure along the way based on the evidences detected in signature plots and in streamflow hydrographs. The resulting model parameterisations at the annual, monthly and daily time scales, are summarised in Table 2.7.

Physical processes of the hydrological cycle do not change with time scale, yet the emergent or dominant processes at the annual, monthly and daily time scales are different. Notably, the parameters used in the models at different time scales must have identical values, and no variations have been allowed to try and improve the model predictions. The small set of parameters used serve as surrogate indicators of emergent properties of the hydrology with a high ability to provide the best information at the catchment scale and evolving time scales. Model parameter values are estimated a priori from available field data, from assessment of observed streamflow recessions or taken from literature, and calibration of model parameters through optimisation in the traditional manner has been avoided. It is expected that the chosen model parameters and input variables may be available also in regions with less data and lower monitoring standards.

This study concentrated on the reduction of structural uncertainty in the formulation of appropriate models for estimating the water balance at different time scales but the successful performance of the models also rely strongly on the quality of parameter sets and input data. With parsimonious water balance models one can at the very least concentrate more efforts on the estimation of a smaller set of parameter and input data than it would otherwise be the case for a larger set with more complex models. However, how to deal effectively with
uncertainties of parameter and input values - measured at the plot scale and regionalized to the catchment scale - is left for future work.

Table 2.7: Components of the model results based on a minimum number of input variables and model parameters (refer to Notation for the description of symbols).

<table>
<thead>
<tr>
<th>Model type</th>
<th>Number and specification of applied input variables</th>
<th>Number and specification of applied parameters</th>
<th>Model results</th>
</tr>
</thead>
<tbody>
<tr>
<td>annual</td>
<td>$2: [P]$ and $[E_p]$</td>
<td>$5: [\bar{m}], [\bar{i}<em>w]$ and $C</em>{ip}$</td>
<td>$[E_a], S$ and $[Q_{se}]$</td>
</tr>
<tr>
<td></td>
<td>${= f(D_{ip}, \phi, \theta_{pwp})}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>monthly</td>
<td>$2: {P}$ and ${E_p}$</td>
<td>$9: [\bar{m}], {\bar{i}<em>w}, C</em>{ip}, \sigma_T, T_{crit}, mf$ and $t_c$</td>
<td>${P_r}, {P_s}, {Q_N}, {E_a}, S_N, S$, ${Q_{se}}$ and ${Q_{ss}}$</td>
</tr>
<tr>
<td>daily</td>
<td>$2: P$ and $E_p$</td>
<td>$8: C_{fc} {= f(D_{ip}, \phi, \theta_{fc})}, C_{ip}, T_{crit}, mf, t_c,in$ and $t_c,bf$</td>
<td>$P_c, P_s, Q_N, E_a, S_N, S, Q_{se}, Q_{in}$ and $Q_{bf}$</td>
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Chapter 3: Water Balance Modelling with Fuzzy Parameterisations: Application to an Alpine Catchment

Keywords:
- annual and monthly water balance
- hydrologic modelling
- fuzzy logic, parameter uncertainty
- fuzzy parameters and climatic inputs
- Upper Enns catchment

3.1 Introduction

Hydrologists often find themselves in situations where they have to determine parameter and input values for hydrological models based upon uncertain information. In many cases of catchment modelling there is no or inadequate information available to estimate the parameter and input values with the kind of precision required of deterministic models. If only data from point observations and measurements are available the modeller faces great difficulty interpreting and using this data for the modelling effort at the catchment scale. The problem of lack of data is especially critical in ungauged catchments since there is no prospect of estimating the parameter values by calibration. Thus methods that permit the incorporation of all sources of information, especially uncertain and heuristic information, would be very valuable towards the development of hydrological predictive models. In recent years calls to include the ability to estimate measures of predictive uncertainty as part of the development of hydrological models have increased, especially in the light of the realisation of “equifinality” of model structures and parameterisations (Beven 2001; Freer et al. 1995). Equifinality is the term given to the phenomenon whereby many or infinite combinations of parameter values (and even model structures) can give rise to the same levels of fit to observed data, making the identification and estimation of parameter values a non-trivial task, especially when models are overly complex. Hydrologists have in the past tended to aim for higher and higher levels of precision in model formulations, parameterisations and computations while ignoring the quality of information that serves as the basis for their inference. It has been suggested that this problem can be addressed more easily with fuzzy logic than with some other methods (Dubois and Prade 1986 1988, Franks and Beven 1999, Franks et al. 1997). There are two alternative approaches for the incorporation and prediction of uncertainty in hydrological models: 1) the traditional stochastic approach based on the treatment of all parameters and variables as random variables, with specified or derived probability distributions; and 2) methods based on fuzzy logic, a radically different approach not based on the treatment of the variables as random variables but as fuzzy numbers, and offering a number of advantages compared to the stochastic approaches. Methods based on the stochastic approach are already widely established in hydrology (Garen and Burges 1981; Gelhar 1986) and interpretation of their results for decision-making is fairly straightforward and well established. However, catchment hydrological systems are extremely complex, comprising non-linear interactions of many
inter-dependent processes. Consequently, stochastic approaches have the disadvantage that the mapping of uncertainties between inputs and parameters to the outputs cannot be done analytically, and can only be done through Monte Carlo procedures, which are computationally intensive. Stochastic methods have the additional disadvantage that with the information currently available, it is not possible, or at the very least not easy, to completely prescribe the probability density functions of all of the inputs and parameter values.

As opposed to the difficulty in estimating precise values of catchment properties, model parameters or their probability distributions, hydrologists, with experience, should be in a better position to specify the intervals within which the catchment properties, and hence the model parameters, may lie. In the fuzzy water balance model presented in this chapter, fuzzy membership functions are used to characterise catchment physiographic properties, and consequently parameter values, and model predictions of hydrologic state variables and fluxes are also presented in terms of fuzzy membership functions.

Despite the paucity of actual measured databases from which probability distributions can be estimated for various catchment characteristics and climatic inputs, much heuristic information is available and can be supplied by "experts" who have worked in the respective regions. The fuzzy approach has the advantage that it can deal with any available quantitative information, and can also incorporate qualitative and heuristic information (Türksen 1991; Civanlar and Trussel 1986; Hisdal 1994). Computations within the fuzzy framework, using the rules of fuzzy arithmetic, are easy and straightforward. The fuzzy approach has the additional advantage that the model outputs are characterised by grades of fuzzy membership, expressing the reliability of the model predictions, thus leading to a realistic estimation of predictive uncertainty. The theory of fuzzy systems on which this method is based (Zimmermann 1995) can therefore bring us one step closer to resolving a hydrological dilemma, namely, about how to develop models of a complexity commensurate with the quality of, and uncertainty in, meteorological input data and the catchment’s physiographic properties.

On the other hand, fuzzy approaches have the disadvantage that the interpretation of the results coming from analyses and simulations based on fuzzy logic are yet not widely accepted or understood in natural sciences and in engineering practice. Because of the inability to convert fuzzy predictions into probability distribution functions of model outputs, engineers tend to show a preference for traditional stochastic methods in spite of their inherent disadvantages discussed before.

**Scientific Objectives**

In this chapter an application of the fuzzy modelling approach is present to the Upper Enns catchment in central Austria. Based on fuzzy temperature and precipitation time series, and fuzzy parameter values relating to catchment physical properties, and fuzzy climatic inputs, the runoff components including surface flow, interflow and baseflow, as well as snow cover, evapotranspiration and soil water storage, are estimated at time scales ranging from a day to the whole year. The key scientific objectives of the study are, briefly:

- To present the formulation of a lumped water balance model in the framework of fuzzy logic;
- To demonstrate the ability of a fuzzy model to quantify the relative importance of various parameters and input values; and,
- To demonstrate the interconnections between model complexity, predictive uncertainty, and accuracy of predictions

In this respect, the present study does not focus on the uncertainties inherent in the model conceptualisation, rather it builds on the results of Chapter 2. This chapter represents merely an initial application of the fuzzy logic
methodology to the Upper Enns catchment. While the aim of this study is to present a more general idea of fuzzy water balance modelling, there exists considerable potential for further advances in the use of fuzzy techniques. The present study is restricted to illustrating the application of fuzzy logic to quantify the relative impact of uncertainty estimates of model parameters and climatic inputs, and of model structural complexity, on the fuzziness of model predictions. The extension of the model to address the more general problem of prediction of ungauged catchments is left for future work.

Outline of the Chapter
This chapter begins with a discussion of the sources of uncertainty and imprecision in hydrological modelling, followed by a brief, basic introduction to the theory of fuzzy logic. Next the water balance model is presented, at the daily time step, that has been developed previously in Chapter 2 for this catchment.

A state-space approach (Özelkan and Duckstein 2001) is adopted for the presentation of the model equations, and the model is then expressed in a fuzzy framework. Due to fuzzy parameters and climatic inputs all state variables or outputs will also be fuzzy, and the state transition and output functions can be constructed through rules of fuzzy arithmetic. Based on analysis of all available information, the formulation of the parameters and input variables of this model for this catchment is then described, within the new fuzzy framework, in terms of fuzzy membership functions. Following this section, applications of the fuzzy water balance model to this catchment are presented, and the model is utilised, through systematic and step-by-step sensitivity analysis, to investigate the relative importance of model parameters, climatic inputs and model structure (complexity) on the resulting uncertainty of model predictions.

3.2 Uncertainty and Imprecision
Sources of uncertainty in hydrological modelling can be separated into two groups: 1) structural uncertainty, and 2) parameter uncertainty. Structural uncertainty refers to uncertain knowledge of the overall functioning of the catchment and the uncertainty in the model structures used to capture this. Parameter uncertainty refers to the uncertainty of the inputs to, and the parameters of, an assumed model structure.

3.2.1 Structural Uncertainty
We still do not have a perfect understanding of the climate, soil, vegetation and topographic controls on water balance, therefore leading to inadequate representations of catchment water balance in hydrological models. Attempts at applying physically based models at the catchment scale have not been entirely successful due lack of data or to model over-parameterisation. The theories on which these models are based are dependent on small-scale physics, whereas the model applications are often at the catchment scale. A large number of conceptual models have been built, and their performances evaluated. Alley (1984), Franchini and Pacciani (1991) and Chiew et al. (1993) compared a number of frequently applied water balance models of different levels of complexity. Their conclusions are clear and consistent. For example, Alley (1984) stated that simulated values of a state variable, such as soil moisture storage, differ strongly amongst models with optimised parameters, sometimes indicating an entirely different type of basin response to precipitation. Therefore the physical appropriateness of many model structures and parameters, as well as that of the assumed state variables has been
unclear. Often many different models are shown to produce similar fits to observed data, thus leading to structural uncertainty.

Model development requires the support of data analysis, interpretation, hypothesis testing and the reconciliation of model concepts with field observation (Beck 1994). The distillation of hypothesis is essential to model formulation and may help eliminate unsound preconceptions more swiftly than might otherwise be the case (Wheater and Beck 1995; Eder et al. 2002). Yet mostly this is not the way model structures are chosen or adopted. Instead, they are often chosen arbitrarily, the choices made a priori based on previous system knowledge, or on model structures adopted by others.

In Chapter 2 an alternative, data driven, “downward” approach to model conceptualisation was presented. The resulting, insightful definition of the model structures based on emergent properties had helped in the formulation of sound and parsimonious models with parameters mostly estimated a priori and giving good predictions.

3.2.2 Parameter Uncertainty

Considerable uncertainty can be expected in the estimation of climatic input variables (i.e., precipitation, temperature) (Skoda 1993a), of the observations with which the model predictions (i.e., discharge) are evaluated, and of the model parameter values (i.e., soil moisture capacity). As in all natural sciences, it is difficult to express the physical, hydrological and meteorological variables by crisp measures. There are likely to be serious uncertainties in determining these properties even at the point or plot scale. The spatial extrapolation, or scaling up, of information from the point or small plot scale to the catchment scale is an additional difficulty, and the resulting uncertainty tends to get worse the more diverse the monitoring (point or plot) and modelling (catchment) scales are. In this regard, one can talk of random and systematic errors inherent in the observations, and errors associated with the scaling up of information from the point or plot (observation) scale to the catchment (or modelling) scale.

3.2.2.1 Random Errors

The random error in the act of observation may occur due to unnoticed alteration of the standardised measurement condition. For example when measuring precipitation, deviations may be caused, for instance, by the blocking of the drainage mechanism of the rain gauge, accidentally incorrect reading, and confusion during the date registration procedure. Sometimes these data errors are filtered out. Data uncertainties arising from random errors are not included in the estimation of fuzzy model inputs and parameter values used this study, but can be easily included in subsequent extensions of this study.

3.2.2.2 Systematic Errors

Systematic errors can be caused by specific measurement and computational techniques. Such errors arise when applying a rating curve prepared before a devastating flood event to the river water levels in the post-flood situation, and thus not accounting for changes to the riverbed morphology. Errors in precipitation measurements can occur, especially in high elevation zones, due to strong winds causing precipitation to be blown past the gage (Sevruk 1986).
Another type of error may occur due to the spatial extrapolation of point observations from a monitoring network with a density and distribution that are not sufficient to fully capture the spatial variability of parameters or variables. Especially in Alpine regions the density and location of point observations are of great importance in order to represent the very diverse topographic and local meteorological characteristics. In general, micro- to meso-scale variations of climatic features and soil properties increase with increasing topographic complexity of the terrain (Christakos 1992). The placement of observation sites would have to be denser in mountainous regions than in flat areas, in order to attain the same quality in spatial extrapolations of the observations. But the opposite is in fact true, as detailed information, for instance, on soil characteristics is mainly mapped in intensively used agricultural regions in the lower valley zones but very sparsely in high elevation zones. The same also applies to the network of climate monitoring stations. The accuracy of various interpolation techniques strongly depends on the positioning of monitoring stations. Using data from monitoring stations at locations that do not account for the spatial characteristics of the study area will introduce uncertainty in the regionalized estimates. The quality of water balance estimates relies heavily on the evaluation period of the meteorological observations. These have to be carried out over many years (to produce sufficiently long time series), according to standardised rules for instruments and observations with largely unchanged local conditions during the reference period, for us to have confidence in predictions of water balance into the future.

Later on in this chapter more light will be shed on the uncertainties involved in the estimation of parameter and input values that are used in the water balance model. These estimations of input data and parameter values, and their associated uncertainty measures, serve as basis for the specification of fuzzy membership functions.

### 3.2.3 Expert Information

As mentioned above, point measurements or estimates of physical and meteorological variables are associated with inherent errors or uncertainties. The up-scaling procedures for scaling up from the point or plot scale to the catchment scale introduce additional uncertainties, governed by the adequacy of the spatial extrapolation procedure (mainly interpolation) used in the model. The interpretation or processing of point observations for obtaining the corresponding estimates at the catchment scale often rely on the experiences and intuitive feel of experts about the general catchment characteristics and climate, about the small-scale variability of catchment properties, and meteorological variables.

It turns out that methods based on fuzzy logic allow us to map the expressions of expert judgement, expressed verbally in heuristic terms, in a mathematically useful manner without loosing the character of imprecision. They can be used to incorporate both the uncertainties inherent in traditional estimates of catchment properties, as well as the imprecision of heuristic statements and estimations made by experts. These methods are discussed in the next section where the basic principles of fuzzy theory are outlined in the context of the water balance modelling to be presented later on in this chapter.

### 3.3 Fuzzy Membership Functions

Zadeh (1965) first defined a fuzzy set by generalising the mathematical concept of an ordinary set. From those early days applications of this new concept of uncertainty has been successful in a wide range of topics. In this section the generalities of the relevant fuzzy theory is briefly introduced while the associated basic arithmetic
A fuzzy number may be considered as an extension of the concept of the interval of confidence. Instead of defining just one interval of confidence, the latter is considered at several levels, which may be called levels of presumption expressed by $\mu$. From the lowest ($\mu = 0$) to the highest level of presumption ($\mu = 1$) multiple levels of presumption may be defined. Continuing with the example of soil depth, in general, one can say that the larger one defines the interval for the unknown mean soil depth the smaller is the level of presumption in making that statement (e.g. $\mu = 0$). Conversely, with a smaller interval of confidence the level of presumption is higher, the highest level of presumption being attained when the interval of confidence shrinks to a single crisp value, say 200 mm for $\mu = 1$. For example, the mean catchment soil depth may be expressed using a triangular function, relating interval of confidence to level of presumption. The interval of confidence at the lowest levels of presumption ($\mu = 0$) can be defined as [100 mm, 300 mm], this being a very conservative estimate. On the other hand, say, one estimates the mean profile depth as 200 mm for $\mu = 1$. One can thus relate the interval of confidence to multiple levels of presumption that lie in the range [0, 1] – this is called the fuzzy membership function. For a continuous transition of the level of presumption $\mu$ in the range [0, 1], any finite number of characteristic values, or a continuous function, can be defined. An example of the resulting fuzzy membership function of the mean catchment soil depth is presented in Figure 3.1. Note that the theory of fuzzy sets should not be confused with the theory of probability, and a fuzzy set is not a random variable. A fuzzy set is merely an extrapolation of the concept of the interval of confidence to multiple levels of presumption in the range [0, 1].
Defining formally, \( \hat{D} \) is called a fuzzy set of a referential set, for example \( R \), if the set consists of ordered pairs such that

\[
\hat{D} = \{ (d, \mu_d(d)) : \forall d \in R, \mu_d(d) \in [0, 1] \} \tag{3.1}
\]

Again referring to the example cited above \( d \) may be the suspected soil depth at multiple levels of presumption within \([0, 1] \), whereas \( \hat{D} \) is the membership function of the estimated mean soil depth. A fuzzy number is defined as a spatial case of a fuzzy set, a fuzzy subset, which is based on the coupled concepts of the level of presumption and interval of confidence.

![Figure 3.1: A binary set, a non-normal and non-convex fuzzy set, and a triangular fuzzy number.](image)

A fuzzy number \( \hat{D} \) in \( R \) is a fuzzy subset in \( R \) that is convex and normal (Kaufmann and Gupta 1991) (Figure 3.1). Normality in the context of fuzzy sets means that there exists at least one value of \( d \in \hat{D} \) such that \( \mu_d(d) = 1 \). A fuzzy set is (quasi) convex if the membership function of \( \hat{D}, \mu_d(d) \in [0, 1] \), does not show a local extreme. Consequently the membership function of \( \hat{D} \) is always non-decreasing on the left of the peak, and non-increasing on the right of the peak.

A triangular fuzzy number is a special type of a fuzzy number with two linear functions on either sides of the peak. Left-right symmetry is not a necessary condition for a triangular fuzzy number. A simple method of defining a triangular fuzzy number is by assessing the symmetric or semi-symmetric membership function with three points (Dubois and Prade 1980), as generally used in this study:

\( \hat{D} = (d^L, d^C, d^R) = (100\text{mm}, 200\text{mm}, 300\text{mm}) \).

Of course, any crisp number can be defined as a triangular fuzzy number with \( d^L = d^C = d^R : \hat{D} = (200\text{mm}, 200\text{mm}, 200\text{mm}) \).

### 3.4 Basic Model Construct

The water balance model used here is a lumped conceptual model, based on a daily time step, developed and tested previously in Chapter 2 for the Upper Enns catchment. The model incorporates the processes of runoff...
generation by the mechanisms of saturation overland flow (whenever soil moisture storage capacity is exceeded, equivalent to total bucket capacity), interflow (shallow subsurface flow) whenever soil moisture storage exceeds the limited storage capacity corresponding to the soil’s field capacity, and deep groundwater flow (or baseflow). Evapotranspiration is simply assumed to be equal to the potential evapotranspiration estimated by the Thornthwaite method when periods free of precipitation are being considered and otherwise, during precipitation events, fluxes of evapotranspiration are assumed to be zero. Precipitation is partitioned into rainfall or snowfall based on a single threshold air temperature, and snowfall accumulates into a snowpack during the winter months. The model applies a temperature – index algorithm for simulating snow processes. The rate and timing of the snowmelt process are estimated based on the same threshold air temperature, and a fixed melt factor.

The model structure is presented schematically in Figure 3.2. The water balance dynamics of the catchment is characterised by the two coupled equations, involving two state variables representing soil moisture storage (Equation 2.18) and snow water storage (Equation 2.15), respectively (refer to the Notation provided in the front section of this book for a list of abbreviations and their brief descriptions).

This version of the model uses eight parameters, all of which were estimated a priori for the Upper Enns catchment; the physical meanings of these parameters and the details of their estimation are provided in Chapter 2. The input data required for the running of the model are $P$ and $E_p$. The model uses the parameters $C_{fp}$, a function of $(D_{ep}, \theta_f, \theta_{wp})$, $C_{ip}$, a function of $(D_{ip}, \phi, \theta_{wp})$, $T_{crit}$, $mf$, $t_{c,in}$ and $t_{c,bf}$.

Precipitation falling as snow $\hat{P}_s$ or as rain $\hat{P}_r$, snowmelt runoff $\hat{Q}_N$, snow water equivalent in the snowpack $\hat{S}_N$, actual evapotranspiration $\hat{E}_a$, total soil moisture capacity of the soil profile above the permanent wilting point $\hat{C}_{ip}$, soil moisture capacity until field capacity $\hat{C}_{fp}$, soil moisture storage level $\hat{S}$, saturation excess runoff $\hat{Q}_{se}$, catchment response time $t_{c,in}$ of interflow $\hat{Q}_{in}$, and catchment response time $t_{c,bf}$ of the baseflow component $\hat{Q}_{bf}$. All input data and parameter values are fuzzy.

Figure 3.2: Fuzzy water balance model concept accounting for snowmelt runoff, saturation excess runoff, interflow and base flow.
In this chapter, this model is recast from its formerly deterministic form into a new, fuzzified form, based on the types of fuzzy membership functions mentioned above and associated rules of fuzzy arithmetic briefly summarised in Appendix 1. With the change to fuzzy form, the model now uses fuzzy input data and parameter values (Figure 3.2), and in turn produces various time series of fuzzy model outputs (fluxes) and system states (Figure 3.3). These are: $P_r$, $P_s$, $Q_N$, $S_N$, $E_a$, $S$, $Q_w$, $Q_{in}$, and $Q_{bf}$.

The justification for the choice of membership functions for the various fuzzy parameters and input data is described next.

Figure 3.3: The fuzzy water balance model approach: fuzzy input data and fuzzy parameters result in fuzzy outputs. Carry-over of fuzzy system states from time $t$ to $t+1$.

### 3.5 Estimation of Fuzzy Parameters and Inputs

Due to uncertainties in the observation of various point data, and due to insufficient knowledge about the spatial distribution of catchment physiographic properties and climate inputs, the estimation of mean catchment properties and climatic variables is difficult, leading to uncertainty in the specification of model parameters and climatic inputs. In this chapter, such uncertainties are expressed by means of fuzzy membership functions for each of the model parameter values and climatic inputs.

It was chosen to use triangular membership functions to describe the fuzziness of all parameters and climate inputs used in the model. Triangular membership functions are the simplest to use, without considering any other shapes of membership functions. Mathematical operations with triangular fuzzy numbers always result in fuzzy numbers, which are not necessarily triangular any more, but still retain a unique maximum at the highest level of presumption. Computational results obtained from the fuzzy water balance model for the highest level of presumption are therefore identical to results of a conventional (deterministic) water balance model of the same structure.
3.5.1 Climatic Inputs: Precipitation and Temperature Time Series

The accuracy of precipitation measurements has been discussed over many years in some European countries, and significant progress has been achieved to correct precipitation data for systematic errors (Sevruk 1986; Skoda 1993a). In general, however, data users are still provided with uncorrected precipitation measurements. Recommendations on the correction of precipitation data are limited in Austria, especially for the Upper Enns catchment. Thus the Swiss (Sevruk 1983; 1986) and the German (Richter 1995) experiences are combined, with what little of the Austrian experience is available, to define the membership functions of fuzzy precipitation estimates that will serve as inputs to the water balance model. Estimates of the average correction factors for the hydrological year (October to August) range between a minimum of 10% and a maximum of over 25% in the highest elevations of Switzerland (Sevruk 1986). The seasonal variation of the correction factors, for instance at the monitoring station at Davos (1580 m a.s.l.), a station close to Austria and with similar meteorological features to those of the Upper Enns catchment, fluctuate almost sinusoidally between about 30% in March and about 8% in July.

As a first approximation, the regionalized daily precipitation estimates are defined as triangular fuzzy numbers (Table 3.1) – the centre values of the triangular fuzzy numbers are set to the uncorrected precipitation estimates, while the right and left values correspond to correction factors of +20% and −2%, respectively. The highly asymmetric shape of the triangular fuzzy numbers for precipitation data signifies that over-estimation of catchment average precipitation is thought to be highly unrealistic; this is because precipitation measurements generally under-estimate the true precipitation volumes (Sevruk 1986).

Compared to precipitation measurements, point measurements of temperature are much more accurate and the spatial coverage with 12 temperature stations in the Upper Enns catchment is relatively dense. Deviations of the mean regional temperature estimates from the true values are considered to be low. Fuzzy numbers of temperature are defined on more realistic uncertainty measures. Neither under- nor over-estimation of temperature could be found to be more evident, and hence a symmetrical, triangular fuzzy membership function with left and right values deviating with ±1.5 °C from the centre value (Table 3.1) was adopted, as a first approximation, with the centre value taken to be equal to the kriged (area-averaged) value.

Based on these temperature values the fuzzy magnitude of the potential evapotranspiration was computed using the classical Thornthwaite equation (Thornthwaite 1948).

<table>
<thead>
<tr>
<th>Fuzzy Model</th>
<th>Input Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{T}$ [°C]</td>
<td>($\bar{T} + 1.5$, $\bar{T}$, $\bar{T} - 1.5$)</td>
</tr>
<tr>
<td>$\bar{P}$ [mm]</td>
<td>(0.98 $\bar{P}$, $\bar{P}$, 1.20 $\bar{P}$)</td>
</tr>
<tr>
<td>Fuzzy Model</td>
<td>Parameter</td>
</tr>
<tr>
<td>$\bar{m}_f$ [mm/K/d]</td>
<td>(0, 1.5, 3)</td>
</tr>
<tr>
<td>$\bar{E}_{rev}$ [°C]</td>
<td>(0, 1, 2)</td>
</tr>
<tr>
<td>$\bar{C}_p$ [mm]</td>
<td>(250, 310, 370)</td>
</tr>
<tr>
<td>$\bar{C}_f$ [mm]</td>
<td>(100, 125, 150)</td>
</tr>
<tr>
<td>$\bar{t}_{cv,jh}$ [day]</td>
<td>(7, 13, 19)</td>
</tr>
<tr>
<td>$\bar{t}_{c,vj}$ [day]</td>
<td>(40, 46, 51)</td>
</tr>
</tbody>
</table>
3.5.2 Differentiation between Rain and Snow, Snowmelt and Accumulation

The snowmelt algorithm is based on the use of two parameters: a melt factor $mf$ and the critical air temperature $T_{crit}$. In Chapter 2 the melt factor $mf$ was estimated on the basis of the calculated short and long wave radiation values, and considering the catchment's topographic features, cloud cover, vegetation cover, and estimated emissivity of air. The results were similar to those obtained in previous studies on the Enns catchment by Nachtnebel et al. (1993) and Fuchs (1998), to those of Braun (1985) who analysed snowmelt characteristics of various catchments in Germany, and to those obtained from Lauscher's work (1982) in the Vienna region. The melt factor, $mf$, and the threshold air temperature, $T_{crit}$, exhibit temporal variations within the day, as well as through the whole winter season, and also a spatial variability that strongly depends on topographic features. Uncertainties in the estimation of these parameters can never be fully resolved. The estimates of the fuzzy parameter values presented in Table 3.1 are the best possible estimates obtained from their variation between winter seasons of different years and spatial variation throughout the catchment.

3.5.3 Soil Properties

Even though numerous field measurements on soil properties such as soil depth and porosity are available on the plot scale within the study region it is difficult to determine crisp mean values at the catchment scale. The predominant soil profiles are Histosols, Gleysols, Fluvisol, Lithosols, Podsol, Cambisol, Rendzina and with spatial coverage of approximately 1%, 3%, 3%, 4%, 5%, 22%, 25% and 36% of the total catchment area, respectively. Approximately 1% of the study area shows bare rock, located in the highest elevations at or around the mountaintops.

Estimated soil properties of all soil types predominant in the Upper Enns catchment are presented in detail in Chapter 2. In the present chapter these estimates are treated as fuzzy numbers and redefined in terms of their membership functions. They are also used to estimate the catchment-scale profile soil moisture storage capacity, $C_{tp}$, and the catchment-scale soil moisture storage capacity up to field capacity, $C_{fc}$, using the following formulae:

\[
C_{tp} = \left( \sum_{i=1}^{I} D_{tp,i} \left( \Phi_{j} \leftarrow \hat{\phi}_{tp,i} \right) \bullet \left( \hat{\phi}_{j} / 100 \right) \right) \quad (3.2a)
\]

\[
C_{fc} = \left( \sum_{i=1}^{I} D_{tp,i} \left( \Phi_{j} \leftarrow \hat{\phi}_{fc,i} \right) \bullet \left( \hat{\phi}_{j} / 100 \right) \right) \quad (3.2b)
\]

$I$...number of different soil types

$C_{tp}$ and $C_{fc}$ are parameters of the lumped, water balance model, both in turn being treated as fuzzy numbers, estimates of which are also presented in Table 3.1.

3.5.4 Catchment Drainage Characteristics

The model incorporates three mechanisms of runoff generation; quick runoff $Q_{se}$, interflow $Q_{in}$ and baseflow $Q_{bf}$. The rates of baseflow and interflow are governed by two characteristic delay times $\hat{t}_{c,bf}$ and $\hat{t}_{c,in}$,
respectively, both of which are also fuzzy parameters of the models. The method of estimation of these drainage parameters, through an inverse procedure from a representative (master) recession curve extracted from available runoff measurements, is described in Wittenberg (1994) and was adopted in Chapter 2 (Eder et al. 2002).

The transition between surface runoff, interflow and baseflow in the streamflow record is not sharp but gradual, and therefore it is very difficult to separate these with a view to estimating the residence times $\hat{t}_{c,bf}$ and $\hat{t}_{c,in}$ unambiguously. In the case of Alpine catchments, it is known that during the winter season, since temperatures remain below freezing and precipitation does not contribute to runoff, the main contributions to discharge mainly come from drainage of groundwater. Consequently the lowest part of the master recession curve extracted from winter low flow records is chosen for the determination of the response time for baseflow, $\hat{t}_{c,bf}$. For the estimation of the delay time for interflow, $\hat{t}_{c,in}$, the upper part of the master recession curve is used, and the non-linear, transition part, lying between these upper (interflow) and lower (baseflow) parts of the master recession curve, is left out of the analysis. The master recession curve is based on flow records only during major breaks in the precipitation record to avoid contamination by flow contributions from surface runoff.

In Chapter 2 these drainage parameters are estimated from master recession curves based on streamflow gauging at Admont (assuming discharge is not measured at Liezen), the closest discharge monitoring station to Liezen. In this chapter, these previous estimates of catchment response times are redefined as centre values of symmetric triangular fuzzy numbers. Left and right values of the triangular fuzzy numbers are defined based on sensitivity analyses of recession curves which considered the uncertainties due to the compilation of the master recession curve using data from different years and to the vague definition of the separation criterion between interflow and baseflow. The transition between surface runoff, interflow and baseflow in the streamflow record is not sharp but gradual, and therefore it is very difficult to separate these with a view to estimating the residence times $\hat{t}_{c,bf}$ and $\hat{t}_{c,in}$ unambiguously. Estimates of fuzzy membership functions are presented in Table 3.1.

### 3.5.5 Discharge

In later sections the simulated monthly and annual discharges are compared with observed discharge measurements. It is a matter of fact that gauged discharge records can also contain inherent measurement errors. The magnitude of error differs for different measuring techniques, is dependent on the discharge volume, might show seasonal variations (i.e. because of vegetation, snow and ice), and the physical positioning of the gauge as well as the maintenance of the instrumentation may have an influence on the quality of measurements. In general, comparison of model predictions and observations should also account for the uncertainty in the discharge measurements. However, as a first step, this is not attempted in this chapter. Rather, the observed discharge values are kept to their crisp estimates as this chapter focuses on the fuzziness of the model predictions only.
3.6 Mathematical Formulation

The soil moisture store is updated daily using the following water balance equation:

\[
S(t+1) = S(t) + P_r(t) \Delta t - E_a(t) \Delta t - Q_{se}(t) \Delta t - Q_{in}(t) \Delta t - Q_{bf}(t) \Delta t - Q_N(t) \Delta t
\]  

(3.3)

Similarly, the snow water equivalent in the snowpack is also updated daily using the following balance equation:

\[
\hat{S}_N(t+1) = \hat{S}_N(t) + \hat{P}_s(t) \Delta t - \hat{Q}_N(t) \Delta t
\]

(3.4)

Note that \(Q_N\) appears in both water balance equations as the melt water is assumed to contribute to the soil water store rather than flowing directly to the river.

**Form of precipitation**

The total daily precipitation depth is partitioned into depths of snow and rainfall according to the critical temperature \(T_{\text{crit}}\) as threshold:

\[
\hat{P}_r = \hat{P} \quad \text{if} \quad \hat{T} > T_{\text{crit}}
\]

\[
\hat{P}_s = \hat{P} \quad \text{if} \quad \hat{T} \leq T_{\text{crit}}
\]

(3.5)

**Conversion from potential to actual evapotranspiration**

\[
\hat{E}_a = \min \left\{ \hat{E}_p; \frac{S}{\Delta t} \right\} \quad \text{if} \quad \hat{P} = 0
\]

\[
\hat{E}_a = 0 \quad \text{if} \quad \hat{P} \neq 0
\]

(3.6)

where \(\hat{E}_p\) is the daily potential evapotranspiration rate, estimated by the Thornthwaite method (Thornthwaite 1948).

**Snowmelt**

\[
\hat{Q}_N = \min \left\{ mf \left( \hat{T} \right); \hat{T} \right\} \quad \text{if} \quad \hat{T} \leq \hat{T}_{\text{crit}}
\]

(3.7)

The snowmelt \(\hat{Q}_N\) is computed using a temperature-index approach (World Meteorological Organization 1986) where \(\hat{T}\) is taken to be a measure of the energy driving the snowmelt to be used in combination with the melt factor \(mf\). \(\hat{T}_{\text{crit}}\) signifies the critical threshold air temperature that has to be exceeded before snowmelt starts and \(\hat{T}\) denotes the mean air temperature within the time interval, here, a day. If \(T\) is below \(\hat{T}_{\text{crit}}\), then \(\hat{T}\) is set to \(\hat{T}_{\text{crit}}\).

**Saturation Excess Runoff**

Saturation excess runoff is produced if and when the net additions to the bucket via precipitation and evaporation are such that the storage of water in the bucket exceeds the capacity of the bucket, denoted by \(\hat{C}_b\). Thus, the rate of runoff generation is given by:
\[
\hat{Q}_{in} (\pm) = \max \left\{ 0; \left[ \hat{S} (-) \hat{C}_f \right] / \Delta t \right\} 
\]

*Interflow*

\[
\hat{Q}_{in} (\pm) = \max \left\{ 0; \left[ \hat{S} (-) \hat{C}_f \right] (\pm) \hat{t}_{in} \right\}
\]

where \( \hat{C}_f \) is the bucket storage capacity until field capacity \( \hat{\theta}_f \).

*Baseflow*

\[
\hat{Q}_{bf} (\pm) = \hat{S} (\pm) \hat{t}_{bf}
\]

The fuzzy arithmetic used in the equations describing the water balance model essentially deal with the mathematical operations of addition, subtraction, multiplication and division, and is summarised briefly in Appendix 1. Other mathematical operations, such as the comparison of two fuzzy numbers (i.e. \( \hat{T} (> \hat{T}_{crit}) \)), are also discussed in Appendix 1.

### 3.6.1 Carry-over of System States

By the rules of fuzzy arithmetic presented in Appendix 1, the magnitudes of the fuzziness of the modelled system state variables (snow water equivalent, \( \hat{S}_N \), and soil moisture, \( \hat{S} \)), computed using Equations 3.3 and 3.4 above, will continue to increase over time. For example, in the case of \( \hat{S}_N \), more and more highly uncertain snow accumulation and depletion processes take place as winter progresses and leads to a continuous increase of fuzziness of \( \hat{S}_N \). In late spring, with rising temperatures \( \hat{S}_N \) reaches zero, hence fuzziness of \( \hat{S}_N \) also vanishes.

Similarly, the uncertainty of simulated soil moisture also increases over time as a consequence of uncertain processes such as for example infiltration, percolation and lateral flows. Fuzziness of soil moisture can diminish only when the soil moisture bucket empties. This is not a realistic possibility in the Upper Enns catchment. When the Upper Enns catchment was modelled, the soil moisture storage (state variable) hardly ever reached zero in the 21-year simulation period, since inputs into the soil moisture bucket are distributed throughout the year (driven by lumped values of precipitation and snowmelt). The fact that the fuzziness of the soil moisture state variable continues to grow with time in an unbounded manner is clearly a disadvantage for lumped fuzzy water balance modelling. Unlike in the case of snow water equivalent, there is no mechanism for the fuzziness of simulated soil moisture values to be brought back to zero from time to time.

One possible approach that enables restricting the fuzziness of modelled soil moisture is to defuzzify (i.e., transformation of a fuzzy number to a crisp representative (Mayer et al. 1993)) the soil moisture storage at each time step before it is carried-over into the next time step. Consequently the fuzziness of the simulated results merely reflects the uncertainty that is introduced into the modelling procedure during one single time step only, not the fuzziness that is carried-over from previous time steps.

In this chapter, it was decided to introduce a formulation that partially accounts for the carry-over of uncertainty from previous time steps. This is done partially because, as indicated before, unconstrained carry-over of fuzziness at each time step would lead to an unbounded increase of the fuzziness of soil moisture storage. In this formulation it is assumed that if the crisp measure of soil moisture storage is equal to the maximum storage capacity of the bucket, then all of the fuzziness is carried-over from \( t-1 \) to \( t \). On the other hand, when
the crisp value of soil moisture storage is zero, then no fuzziness is carried over. In other words, the lower the soil moisture storage the less of the fuzziness of the soil moisture at time step \( t - 1 \) is carried-over to time step \( t \).

In the transition between empty and full model buckets, the extent of carry-over of fuzziness is assumed to be linear.

Fuzziness of \( S \) computed at time step \( t - 1 \) is re-scaled into \( S' \) for input at time \( t \); the carry-over of fuzziness of soil moisture storage is modelled as a function of the so-called soil moisture ratio, \( r_S \):

\[
S'(t) = f(S(t), r_S)
\]

(see Appendix 1 for the mathematical formulation). This formulation of restricting the carry-over of uncertainty is clearly not based on a physical explanation at this stage. The logic behind this formulation is that when the soil moisture storage is high most likely the absolute measure of fuzziness is also high, and \textit{vice versa}.

### 3.7 Annual and Monthly Water Balances

#### 3.7.1 Model Performance with Fuzzy Parameters and Climatic Inputs

The fuzzy water balance model described in the previous section is implemented using fuzzy estimates of daily precipitation, mean daily temperature and physically based model parameters from the Upper Enns catchment over a period of 21 years. The resulting time series of streamflows from the model are processed to generate daily, monthly and annual water balances. In this chapter for the sake of brevity the focus is on annual and monthly model water balances, whereas only statistical measures of the model performance are presented at the daily time step.

In Figure 3.4a monthly time series of fuzzy precipitation and fuzzy potential evapotranspiration (model input), as well as simulated fuzzy estimates of actual evapotranspiration, are presented for the 1975-1979 period. In comparison to time series of crisp values usually presented as a continuous graph, in this case the intervals of confidences of fuzzy values at a certain level of presumption are joined together and are visualised as continuous “ribbons”. In Figure 3.4a and in every following figure, the level of presumption (\( \mu \)) chosen for presentation is 0.8.
Monthly precipitation $\langle \hat{P} \rangle$, monthly potential $\langle \hat{E}_p \rangle$ and actual evapotranspiration $\langle \hat{E}_a \rangle$, monthly snow water equivalent in the snowpack $\langle \hat{S}_N \rangle$ (end of month value), and soil moisture storage level $\langle \hat{S} \rangle$, total soil moisture capacity of the soil profile above the permanent wilting point $\hat{C}_{wp}$, soil moisture capacity until field capacity $\hat{C}_{fc}$, all of which are fuzzy values and standardised by the crisp value of mean annual precipitation $\langle \bar{P} \rangle$. Level of presumption $\hat{\mu}$.

**Figure 3.4:** (a) Fuzzy climatic input variables precipitation and potential evapotranspiration serve as input for the basic water balance model that results in simulated actual evapotranspiration, (b) fuzzy snow cover (water equivalent), and soil moisture presented in relation to fuzzy water holding capacity of the total soil profile and until field capacity.

The parameters governing the water holding capacity of the soil, i.e., $\hat{C}_{wp}$ and $\hat{C}_{fc}$, appear in Figure 3.4b as constant values over time. The time variation of the fuzzy state variables of soil moisture, $\hat{S}(t)$, and snow water equivalent, $\hat{S}_N(t)$, are set in relation to $\hat{C}_{wp}$ and $\hat{C}_{fc}$. The results from the continuous simulation of the water balance model show, in the case of the seasonal accumulation and depletion of snow, that the higher the snow water equivalent the higher also its fuzziness. The same is true for the soil moisture storage.
In order to gain insights into the generation of calculated fuzzy water balance components presented in Figure 3.4 (at $\mu$ of 0.8), the results of one particular day, i.e. 1st of May 1997, are presented in Figure 3.5. On that day in spring 1997 potential evapotranspiration is low as temperature is still low at that time of the year (Figure 3.5a). Precipitation is much higher than potential evapotranspiration on this particular day. The fuzzy numbers of the state variables concerning the soil moisture and the snow storage as well as water holding capacity of the soil profile are presented for the same date in Figure 3.5b. In particular it may be noted that at a high level of presumption, say 0.8, the soil moisture is clearly higher than field capacity, whereas at low levels of presumption the uncertainty of soil moisture increases such that soil moisture could be larger or smaller than field capacity.

![Figure 3.5: Fuzzy membership functions of model input, parameters and system state variables on the 1st of May 1976.](image)

Precipitation $\tilde{P}$, potential $\tilde{E}_p$ and actual evapotranspiration $\tilde{E}_a$, snow water equivalent in the snowpack $\tilde{S}_N$, soil moisture storage level $\tilde{S}$, total soil moisture capacity of the soil profile above the permanent wilting point $\tilde{C}_{wp}$, soil moisture capacity until field capacity $\tilde{C}_{fc}$, all of which are fuzzy values and standardised by the crisp value of mean annual precipitation $\mid \tilde{P}$. Level of presumption $\mu$.
because the interval of confidence for field capacity is located within the interval of confidence for soil moisture. Another interesting point to note is that the fuzzy numbers for potential evapotranspiration, and both storage measures, soil moisture and snow water equivalent, are not triangular fuzzy numbers any more, even though they have been generated on the basis of triangular fuzzy numbers of input and parameter values. Generally May is a month with high snowmelt, which is difficult to predict precisely, i.e. because of fuzzy estimates for temperature and melt factors. Consequently calculated storage values of the snowpack are also fuzzy.

The use of 0.8 as the level of presumption (Figure 3.5) results in specific magnitudes for the intervals of confidence of different fuzzy numbers on any given day. The absolute magnitudes of the intervals of confidences change over time, and these may change seasonally also.

Annual $\bar{Q}$, mean monthly $\langle \bar{Q} \rangle$, and monthly discharge $\langle Q \rangle$ (observed $Q_o$ and modelled discharge $Q_p$), all of which are fuzzy values and standardised by the crisp value of mean annual precipitation $[\bar{P}]$. Level of presumption $\mu$.  

Figure 3.6: Annual and monthly simulated versus observed discharge: Intervals of confidence at the level of presumption of 0.8 generated with the basic water balance model accounting for saturation excess runoff, inter flow and base flow with fuzzy values for climatic input variables and parameters.
The performance of the fuzzy water balance model is presented in terms of characteristic signature plots and hydrographs, which are compared to corresponding graphs based on observed streamflows (Figure 3.6). The parsimonious model, with just eight parameters only, gives good predictions of the signature plots and streamflow hydrographs at the monthly and annual time scales, which was anticipated because of satisfactory results obtained in the previous study of Chapter 2. The crisp graph visualises model results for $\mu$ of 1.0, equivalent to the model results from the deterministic water balance model of the previous study in Chapter 2 based on crisp input data and parameter values. These predictions could be improved with further calibration of the parameter values, but this is not the purpose of this exercise. Rather, the model is used to understand the contributions of the various parameters and inputs to the resulting fuzziness of the model predictions.

The results show that simulated high flows exhibit high fuzziness, whereas low flows are less fuzzy (Figures 3.6c and 3.6d). Monthly results have been aggregated to annual estimates that were presented in Figures 3.6a and 3.6b also for a level of presumption of 0.8. At this stage one has to accept that the fuzzy results are a direct product of the transformation of fuzzy parameters and input data through the water balance model (Table 3.1). Next the relative sensitivity of parameter estimates and the set of input data on the overall fuzziness of simulated discharges is investigated.

### 3.7.2 Model Accuracy and Sensitivity to Fuzzy Parameters and Climatic Inputs

The fuzzy climatic input variables introduce much less uncertainty than the eight fuzzy model parameters. This is demonstrated in Figures 3.7a-d, which presents the simulated fuzzy discharge generated using fuzzy input data (precipitation and temperature) but crisp (defuzzyfied) estimates of parameter values. This suggests that bigger contribution to uncertainty in the predictions of the water balance model comes from fuzziness of the model parameters than from fuzziness of the meteorological inputs because the absolute magnitudes of values and the ranges of the fuzzy numbers of the input data are smaller than those of the model parameters.

Next a possible ranking of the various parameters in terms of their individual contributions to the overall uncertainty of simulated model results is investigated. This was achieved by repeatedly running the model with each parameter held constant (crisp), while letting all other parameters to remain fuzzy and estimating overall, bulk measures of uncertainty. The following ranking of parameters and climatic input variables, ranging from high to low contributions to total uncertainty of the simulated discharge, was finally achieved: $\tilde{C}_{fc}$, $\tilde{t}_{c,ja}$, $\tilde{t}_{c,bf}$, $\tilde{T}$, $\tilde{P}$, $\tilde{T}_{crit}$, $\tilde{m}_f$, and $\tilde{C}_p$. 

61
Annual mean monthly and monthly discharge \( \bar{Q} \) (observed \( Q_o \) and modelled discharge \( Q_p \)), all of which are fuzzy values and standardised by the crisp value of mean annual precipitation \( \bar{P} \). Level of presumption \( \mu \).

Figure 3.7: Performance of the basic water balance model with fuzzy input data but crisp (defuzzyfied) estimates for model parameters.

Table 3.2 lists all parameters and input variables with their potential to overall model uncertainty. Here, the fuzziness of model predictions is evaluated through two criteria, the mean \( \langle \bar{Q} \rangle \) and mean squared \( \langle \bar{Q}^2 \rangle \) of the absolute magnitude of the interval of confidence of simulated daily discharge at the level of presumption of 0.8. If a parameter or input variable is set to a crisp value and both evaluation criteria show high deviations to the initial case when all input variables and parameters are fuzzy then the influence on the overall model uncertainty is larger than in other cases. Among all factors the parameter accounting for field capacity \( C_{fc} \) has the largest contribution to the uncertainty of model predictions. The next two in the list, in terms of their contributions, are the runoff delay times for interflow \( t_{c,ir} \), and baseflow \( t_{c,bf} \). These are followed in order by the climatic inputs.
of $\tilde{T}$, $\tilde{P}$, followed by parameters relating to snowmelt, namely, $\tilde{T}_{\text{act}}$, $\tilde{mf}$ and $\tilde{C}_{ip}$. These results can be potentially useful to improve the estimation of the most influential parameters, or to make modifications to the model structure to avoid the use of parameters that are difficult to estimate, or those that contribute the most to the uncertainty of model predictions. This is investigated next.

Table 3.2. Predictive uncertainty that is dependent on fuzzy input data and parameters for the 1972-1993 period.

<table>
<thead>
<tr>
<th>Estimation of climatic input and parameter values: crisp or fuzzy</th>
<th>$\mu$</th>
<th>$\sigma_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>all fuzzy</td>
<td>0.956</td>
<td>0.076</td>
</tr>
<tr>
<td>all fuzzy except $P$</td>
<td>0.720</td>
<td>0.043</td>
</tr>
<tr>
<td>all fuzzy except $T$</td>
<td>0.716</td>
<td>0.043</td>
</tr>
<tr>
<td>all fuzzy except $C_{ip}$</td>
<td>0.743</td>
<td>0.046</td>
</tr>
<tr>
<td>all fuzzy except $C_{fc}$</td>
<td>0.577</td>
<td>0.028</td>
</tr>
<tr>
<td>all fuzzy except $t_{c, in}$</td>
<td>0.603</td>
<td>0.030</td>
</tr>
<tr>
<td>all fuzzy except $t_{c, bf}$</td>
<td>0.652</td>
<td>0.035</td>
</tr>
<tr>
<td>all fuzzy except $mf$</td>
<td>0.752</td>
<td>0.047</td>
</tr>
<tr>
<td>all fuzzy except $T_{\text{crit}}$</td>
<td>0.737</td>
<td>0.045</td>
</tr>
</tbody>
</table>

Mean absolute ($\mu$) and mean squared magnitude of the intervals of confidence ($\sigma_\mu^2$) of simulated daily discharge ($\tilde{Q}_p / [\tilde{P}]$) at a selected level of presumption $\mu$ of 0.80.

3.7.3 Reducing Model Complexity

The estimation of the soil moisture capacity, $\tilde{C}_{fc}$, is highly fuzzy due to difficulties in estimating soil depth, soil porosity, field capacity and permanent wilting point. Model simulations also demonstrated that $\tilde{C}_{fc}$ also contributed the most to the fuzziness of the model predictions. Similarly, the estimation of delay times for sub-surface flow paths, $\tilde{t}_{c, bf}$ and $\tilde{t}_{c, in}$, are also fuzzy due to the lack of detailed information on sub-surface flow properties of the soils. A reduction of the levels of fuzziness of these parameters is not feasible unless significant advances are made in the spatial estimation of the soil properties. Therefore an alternative route is explored in order to reduce the uncertainty in model results – in this case it is investigated if simplification of the model can lead to more robust models, namely, models with less uncertain predictions.

Collapsing the two sub-surface flow components, $\tilde{Q}_{in}$ and $\tilde{Q}_{bf}$, into just one results in a simpler model concept with a single sub-surface flow component, $\tilde{Q}_{ss}$, is a possibility to reduce the number of parameters, especially the three most fuzzy and influential, namely $\tilde{C}_{fc}$, $\tilde{t}_{c, bf}$ and $\tilde{t}_{c, in}$. This alternative model structure thus accounts for just two runoff components, saturation excess runoff $\tilde{Q}_{se}$ and subsurface runoff $\tilde{Q}_{ss}$. The water balance model for the soil moisture storage, Equation 3.3, is accordingly modified, and is reproduced below:

$$S(t + 1) = S(t) + \tilde{P}(t) \Delta t - \tilde{E}_s(t) \Delta t - \tilde{Q}_{se}(t) \Delta t - \tilde{Q}_{ss}(t) \Delta t$$

(3.11)

The convention for the carry-over of fuzzy soil moisture states described in Appendix 1 applies here, too. The subsurface flow is expressed as:
\[
\hat{Q}_{st} (= S / \hat{t}_c)
\] (3.12)

In this new simpler model, one soil parameter, \( \hat{C}_{fe} \), is no longer used, and the two drainage parameters, \( \hat{t}_{c,bf} \) and \( \hat{t}_{c,\text{in}} \), are replaced by a single parameter, \( \hat{t}_c \), which can be estimated more unambiguously because a separation criterion is no longer necessary. The catchment response time \( \hat{t}_c \) is estimated from the whole of the master recession curve, being representative of the total subsurface flow. The parameter is again estimated as a triangular fuzzy number with a centre value of 21 days, being the result from the new recession analyses. The width of the fuzzy number is estimated according to the sensitivity of the catchment response time to the selection of flow records from different seasons and years. The triangular membership function \( \hat{t}_c = (15, 21, 27) \) is estimated based on these analyses.

The simple alternative model (Equations 3.4-3.8, 3.11 and 3.12) is based on just six parameters. Simulations by this alternative model (Figure 3.8) are less uncertain than the results generated with the original model (Figure 3.6). On the other hand, the results are similar in magnitude to those generated by the original model with crisp parameter estimates but fuzzy climatic inputs (Figure 3.7).
Figure 3.8: Performance of the alternative water balance model accounting for saturation excess runoff and a single sub-surface flow component based on fuzzy parameters and fuzzy input data.

As demonstrated by Figure 3.8 the uncertainty criteria show a significant reduction of fuzziness with the application of the simpler model for the daily as well as the monthly simulations. On the other hand, the results presented in Figure 3.8c indicate that the low flows and especially also the high flows (peaks) were underpredicted by the model, which correspondences with the findings in Chapter 2.

While in Chapter 2 any judgement on model accuracy was solely based on the visual check of hydrographs and signature plots (the aim was to present the downward approach based on emergent properties), now, beside the visual check, the results of the basic model and the simpler alternative model are also compared through quantitative measures. Table 3.3 displays uncertainty criteria ($\sigma$, $\mu$) and certain accuracy criteria for daily as well as monthly discharges. The performances of the models are evaluated in terms of the match of simulated

<table>
<thead>
<tr>
<th>Simulation $\frac{Q_p}{P}$</th>
<th>Observation $\frac{Q_o}{P}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>interval of confidence for $\mu = 0.8$</td>
<td>and for $\mu = 1.0$</td>
</tr>
</tbody>
</table>

Annual $\bar{Q}$, mean monthly $\bar{Q}$, and monthly discharge $\bar{Q}$ (observed $Q_o$ and modelled discharge $Q_p$), all of which are fuzzy values and standardised by the crisp value of mean annual precipitation $\bar{P}$. Level of presumption $\mu$. 

$\mu$ = 0.8
and observed discharges described by the sum of the mean differences (MD), coefficient of correlation (r), Chiew-McMahon criterion (Chiew and McMahon 1994) (CM) and Nash-Sutcliffe criterion (Nash and Sutcliffe 1970) (NS). Crisp measures of observed discharge and the simulated defuzzified model results, defined as the values at the highest level of presumption (Mayer et al. 1993), are compared.

Table 3.3. Criteria of uncertainty and measures of accuracy for the basic and the alternative water balance model for the 1972-1993 period.

<table>
<thead>
<tr>
<th></th>
<th>Basic model</th>
<th>Eqs. (30.3) to (30.12)</th>
<th>Alternative Model</th>
<th>Eqs. (304) to (30.10), (30.13) and (30.14)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>day</td>
<td>month</td>
<td>day</td>
<td>month</td>
</tr>
<tr>
<td>MD</td>
<td>0.956</td>
<td>0.812</td>
<td>0.472</td>
<td>0.203</td>
</tr>
<tr>
<td>$r^2$</td>
<td>0.076</td>
<td>0.853</td>
<td>0.018</td>
<td>0.868</td>
</tr>
<tr>
<td>CM</td>
<td>63.411</td>
<td>73.042</td>
<td>55.886</td>
<td>71.048</td>
</tr>
<tr>
<td>NS</td>
<td>58.732</td>
<td>73.487</td>
<td>61.521</td>
<td>71.766</td>
</tr>
</tbody>
</table>

Mean absolute and mean squared $r^2$ width of the intervals of confidence of simulated daily $\hat{Q}_p/\bar{P}$ and monthly discharge $\left(\hat{Q}_p/\bar{P}\right)$ at $\mu = 0.80$; model accuracy criteria based on observed $(\hat{Q}_o, \{\hat{Q}_o\})$ versus simulated daily and monthly discharge $(\hat{Q}_p, \{\hat{Q}_p\})$ for $\mu = 0.80$.

The estimated accuracy criteria show a small reduction of model performance with the alternative, simple model structure, compared to that of the original model. In the case of the daily predictions, the Nash-Sutcliffe criterion even suggests a slightly improved performance. In other words, the simplification of the model has led to a significant reduction in predictive uncertainty, while experiencing no apparent reduction in the accuracy of model predictions.

### 3.8 Summary and Conclusions

This chapter has presented the application of a fuzzy, lumped water balance model to the Upper Enns catchment in central Austria. The chapter with references to Appendix 1 covered the basic principles of fuzzy logic and the associated arithmetic and showed how these can be used to construct the fuzzy water balance model. The model is then used, with realistic estimates of the uncertainty of catchment characteristics (for model parameters) and climatic inputs, to estimate the uncertainty of runoff predictions at the annual and monthly time scales. The results are routinely presented in terms of ribbons (or strips or bands) of confidence in the model predictions, which in itself can be extremely valuable if they can be properly interpreted for water resources decision-making.

In addition, the model was used to investigate the relative sensitivity of model predictions, in particular predictions of uncertainty of the catchment water balance, to the uncertainty in the various parameters and climatic inputs. It was found, for example, that the model predictions were much more sensitive to model parameterisations than to climatic inputs. This may be the case in this particular catchment, due to large number of rain gauges that were available, and the consequently relatively small uncertainty in the estimates of catchment average precipitation. The relative importance of the various parameters as well as input variables is
also investigated, and it is found that the field capacity was the most important parameter, followed by the subsurface routing parameters, the climate inputs of temperature and precipitation, the threshold air temperature, melt factor and lastly the bucket capacity. This gives the clue as to which of these parameters must be estimated with more confidence in the future, and how increased confidence in model parameters and inputs will increase the confidence in the model predictions.

Finally, the model was also used to investigate the effect of decreased model complexity on the accuracy and uncertainty of model predictions. To do this, the shallow subsurface flow and baseflow were combined into a single subsurface runoff component, thus saving two of the most significant parameters, and the accuracy and uncertainty of the resulting simplified model were recalculated and compared to those of the original model. The results showed that while model accuracy worsens just slightly the uncertainty actually decreases significantly, suggesting that if only monthly or annual predictions are required, then the simpler model is not only sufficient (based on accuracy), but is indeed preferable (because of improved predictive uncertainty).

Through this simple illustration the utility of the fuzzy logic approach was demonstrated to be integrated into water balance models in a straightforward manner. The fuzzy approach can also be used to investigate the relative importance of various parameters, inputs and processes for the development of comprehensive water balance models, and can indeed be used to simplify the models based on systematic sensitivity analyses of the kind presented in this chapter. This can lead to parsimonious models in the future, and models which are capable of predicting not just the mean response, but also confidence in the predictions, in a straightforward manner, without the excessive computational burden characteristic of traditional stochastic, Monte Carlo procedures.
4.1 Introduction

The aim of this study is to present a suitable methodology for estimating spatially distributed water balance over monthly periods in high mountain catchments, at the maximum appropriate resolution, by application of data sets commonly available for those regions. The hydrological model should be generic for high alpine catchments. Water resource estimation serves as the basis for water utilisation strategies in alpine catchments. The ability to assess water resources on a smaller scale than the catchment scale constitutes a major criterion for regional management. Effective management of water resources in alpine catchments over an area of 100 - 1000 km² requires detailed focus on the spatial distribution of water balance, on a monthly to seasonal time scale. In alpine regions, the most important components of the water balance for general planning purposes include precipitation, evapotranspiration, soil moisture and water storage in the snowpack. Runoff components such as surface runoff, interflow and baseflow are also of interest. There is an evident lack of comprehensive information available on the meteorological and hydrological characteristics of high alpine regions. The density of meteorological monitoring networks decrease with increasing altitude and the knowledge of the physical catchment properties and dominant hydrological processes in mountainous regions is less developed at these higher altitudes. Commonly measured data which is available for alpine catchments includes precipitation, temperature and discharge, obtained from a sparse network of monitoring stations. These provide some approximation of the catchment climate. In general, data on the physiographic features of high mountain catchments is available by extraction from basic records of land cover, soil and geological maps. The lack of detailed spatial information on important catchment features can be supplemented by use of surrogate variables, from topographic maps for example. Additionally, it can be important that heuristic knowledge is used in water balance computations, in order to ensure good performance. The topographic complexity and the variability of physical properties within alpine catchments is high, so that elevation differences may cause large variations in precipitation height, temperature and evapotranspiration, for example. However, it is not yet feasible to model the seasonal water balance of such catchments at a fine resolution based on a regular mesh of grid cells, as is common for distributed modelling packages. Selected components of the water balance (e.g. baseflow) can be estimated with
confidence on a sub-catchment scale, of approximately 100 km², others (e.g. meteorological variables) can be determined at a much finer spatial resolution.

Physically-based models, such as the SHE model (Abbott et al. 1986) cannot easily be extrapolated for use in predictions at larger scales, such as seasonal water balance predictions at the catchment scale (Klemes 1983). Soil-vegetation-atmosphere transfer schemes (Nijssen et al. 1997; Famiglietti and Wood 1994) are difficult to apply in the Alps because of problems associated with the use of large (in relation to the topographic variability) regular grid cells. These cells are incapable of accounting for relationships of hydro-meteorological properties with high gradients and elevation differences. These problems cannot be avoided through reduction of the cell size, as at small spatial scales fluxes between grid cells are significant and are difficult to parameterise. Instead the so-called conceptual models (see Franchini and Pacciani 1991; Singh 1995) are more appropriate, having the potential to provide a more holistic representation of catchment response. Their potential disadvantage is that model structures are often chosen arbitrarily, and the related uncertainties never resolved.

4.2 Water Balance Model Concept

The concept is centred on four factors: (1) the definition of appropriate spatial scales for input and parameter estimations, (2) the computational time step, (3) the mathematical formulation of the hydrological model and (4) the model application

4.2.1 Spatial Modelling Scale

Catchments in mountainous region can be advantageously disaggregated into sub-catchments and sub-regions with homogeneous hydrological characteristics, as dependent on topography (Ambroise et al. 1994; Troch et al. 1993). Since many catchment properties are strongly influenced by topography, with gradient and valley-ridge orientation affecting precipitation, temperature, snowmelt etc., topographic maps offer the most reliable source of spatial information.

The first level of catchment disaggregation hence involves the delineation of sub-catchments above discharge gauging stations using digital elevation models. Secondly, sub-catchments are then partitioned into sub-regions at differing elevation zones and with certain slope aspects, as far as possible with homogenous local climate features and physiographic properties. This delineation according to the governing hydro-meteorological characteristics is in many ways similar to the concept of Hydrological Response Units as it was proposed by Leavesley et. al (1983, 1996) for use in the PRMS and investigated by Flügel (1996).

There is considerable heterogeneity of the hydrological processes occurring within a sub-region. However, as it was pointed out by many researchers dealing with the problem of scales in hydrological modelling (e.g. Beven 2000, Beven 2001), there has to be consistency between process representation and spatial modelling scale. Since the model applied in this study is a conceptual model working with daily time steps, it would certainly not be able to reflect the processes occurring at a higher spatial resolution than the sub-regional scale described above. Grid based modelling of the water balance at high spatial resolution (e.g. Ludwig and Mauser 2000) requires models of high complexity as well as information about the initial and boundary conditions at high resolutions in space and time. In the future, this resolutions might grow with the use of advanced remote sensing techniques (e.g. Bach 1995, Mauser and Schädlich 1998, Mauser et al. 1999).
4.2.2 Computational Time Step: Day

Many hydrological processes take place at a smaller time scale than the month being used for water balance simulation, and rely on meteorological input data at a finer temporal resolution. To properly capture snow accumulation and depletion processes, for example, short intervals of melt associated with short warm spells during the winter will not be evident from monthly data.

Since this study does not focus on computational time steps, the internal computational time step of the hydrological model is daily ($\Delta t = 1$) using readily available daily meteorological data as model inputs. The performance of the water balance model and long-term average results are evaluated and presented on a monthly scale.

4.2.3 Water Balance Approach

In this study, the model concept (Figure 4.1a) has similar features to many other hydrological models. It is believed that detailed mathematical formulations of applied hydrological concepts have already been documented in detail elsewhere, so that they will be touched on only marginally in this chapter, as a general overview. The hydrological model developed for this study is similar especially to the HBV model (Bergström 1992, Bergström 1997, Bergström and Forsman 1973) and the ENNS (Nachtnebel et al. 1993) model. The complexity of the applied water balance model is kept to a minimum, according to an optimal balance between model performance and the number of model parameters. This is achieved by focusing on the dominant properties of the hydrological system in the chosen spatial and temporal frame.
Daily precipitation falling as snow $P_s$, or as rain $P_r$, snowmelt runoff $Q_N$, actual evapotranspiration $E_a$, soil moisture storage level $S$, lateral flow component $Q_{lf}$ above threshold storage level $h$ (surface runoff $Q_{sr}$, interflow $Q_{in}$ and baseflow $Q_{bf}$), percolation into $Q_{p-in}$ and out from the storage module $Q_{p-out}$ (percolating surface runoff to deeper interflow zone $Q_{sr-in}$ and percolation from interflow to baseflow zone $Q_{in-bf}$).

Figure 4.1: (a) Schematic representation of the water balance model based on the (b) bucket concept.

The model represents hydrological processes taking place at the sub-regional scale. The water balance of one sub-catchment results from a spatially weighted sum of the sub-regional results (Figure 4.2). Sub-regions are arranged in parallel and are not conceptually interconnected within one sub-catchment. Sub-catchments may be interconnected in parallel or serially, according to the structure of the river network.
Lumped sub-regional (regions 7-14) estimates of precipitation $P_{7-14}^{r}$ and temperature $T^{r7-14}$. Observed sub-catchment (no. 1) discharge $Q_{c1}$ and modelled sub-catchment (no. 2) discharge $Q_{p2}^{r}$.

**Figure 4.2:** Diagram of the conceptual communication between sub-regions and sub-catchments: Sub-regional meteorological input. Gauged discharge of an adjacent sub-catchment 1 serves as model input for sub-catchment 2.

Different water storage phases are conceptualised by specific modules of the hydrological model (Figure 4.1b): e.g. modules for the simulation of snow processes, soil moisture fluctuation in the upper soil layer, evaporation fluxes, surface runoff, interflow and baseflow. The following sections briefly present the water balance equations for all water storage components of the hydrological system.

**Snow Store**

The snow water equivalent in the snowpack $S_{N}$ is computed with the following water balance equation (Bergström 1976):

$$S_{N}(t + 1) = S_{N}(t) + P_{s}(t)\Delta t + P_{r}(t)\Delta t - Q_{N}(t)\Delta t$$

(4.1)

If precipitation falls as snow $P_{s}$, it is temporarily accumulated in a snowpack and depletes again through snowmelt $Q_{N}$ at snow melt conditions. Rain $P_{r}$, falling on the snow pack might freeze in the snowpack or could enhance the melting process due to its energy input.
Correct determination of precipitation type is critical for successful modelling of snowmelt-runoff, but standard precipitation measurements only record total amounts (such as snow water equivalent in the case of snowfall). Hence a criterion is necessary to separate precipitation type into pure snowfall, mixed snow and rain events or pure rain, which for simplicity is provided by the indicator of air temperature. Transitional air temperature intervals are based on data from monitoring stations including daily air temperature, depth of fresh snow and precipitation levels. A density value, derived from daily measurements of fresh snow depths and the total amount of precipitation, is calculated and plotted against the mean air temperature. The lower temperature limit \( T_{\text{min}} \) is given where the calculated density remains below 0.1 \([\text{kg/l}]\), approximately equivalent to that of a newly formed snowpack. The density values increase with mixed precipitation events, with a growing percentage of rain, up to a limit of 1.0 \([\text{kg/l}]\) per day, above which only pure rain is recorded and the upper temperature limit \( T_{\text{max}} \) is set.

Clearly, there is not enough data to apply a comprehensive energy balance model for predicting the spatially distributed accumulation and depletion of a snowpack (Blöschl et al. 1991). A model for the simulation of monthly water balance using only precipitation, and air temperature as a surrogate variable, performs well for the processes steering snow accumulation and depletion. The selected snowmelt model is based on algorithms developed by Anderson (1973) for the NWSRFS snow accumulation and ablation model. Anderson’s model combines a temperature-index method during radiation-melt (non-rain periods) and a more detailed calculation of individual energy fluxes during advection-melt (rain periods).

During rain periods, snowmelt \( Q_{N,\text{wet}} \) in \([\text{mm/} \Delta t]\) is computed from an energy balance equation:

\[
Q_{N,\text{wet}} = NM + EM + HM + RM
\]  (4.2)

The snowmelt calculation takes the net radiative heat transfer \( (NM) \), the turbulent fluxes of sensible \( (EM) \) and latent heat and \( (HM) \) and the sensible heat of the rain water \( (RM) \) into account (see notation section). Assumptions are made concerning the meteorological conditions during rain periods so that various components can be accurately computed from only precipitation and temperature data.

During non-rain periods, melt \( Q_{N,\text{dry}} \) in \([\text{mm/} \Delta t]\) is computed by using a temperature index approach (World Meteorological Organisation 1986):

\[
Q_{N,\text{dry}} = mf \cdot (T_{\text{pos}} - T_{\text{crit}})
\]  (4.3)

In Equation 4.3 \( mf \) refers to the seasonally varying melt factor in \([\text{mm/K/} \Delta t]\) on a certain day of the year (refer to notation section for \( mf_{\text{min}} \) and \( mf_{\text{max}} \)). \( T_{\text{crit}} \) in \([\text{°C}]\) signifies the critical threshold air temperature, that has to be exceeded before snowmelt starts and \( T_{\text{pos}} \) denotes the mean air temperature within the time interval in \([\text{°C}]\). If \( T_{\text{pos}} \) is below \( T_{\text{crit}} \) then \( T_{\text{pos}} \) is set to \( T_{\text{crit}} \).

Apart from the seasonal variation of the melting factor, ranging from the peak winter season (\( m f_{\text{min}} \) on December 21\(^{st}\)) to the beginning of the summer season (\( m f_{\text{max}} \) on June 21\(^{st}\)), the melting factor is also highly dependent on regional catchment characteristics including elevation, aspect, slope and vegetation density. This study presents an approach whereby distributed values of melting factors are derived from \textit{a priori} catchment information, such as location (in geographical co-ordinates), topographic features from a digital elevation model and land cover data from a digital vegetation map. For the estimation of other less sensitive parameters,
assumptions were made on the basis of physical considerations or findings in the above literature. Lumped catchment estimates were used for most of these snow module parameters.

Assuming that \( T_{\text{crit}} \) is close to 0 °C in Equation 4.3, the regional melting factor \( mf \) with units \([\text{mm/K/d}]\), for a certain day and regional physiographic situation, can be expressed as:

\[
mf = \frac{1}{(\rho_w \cdot Lf)} \left( R_{\text{sn}} + R_{\text{ln}} \right) \cdot 1000 \tag{4.4}
\]

where \( Lf \) denotes the latent heat of fusion in \([\text{J/kg}]\) and \( \rho_w \) stands for the density of water in \([\text{kg/m}^3]\). The energy available for snowmelt during rain-free periods (good weather situations) is dominated by the net short-wave heat of radiation absorbed by snow \( R_{\text{sn}} \) and the net long-wave heat of radiation exchange at the snow-air interface \( R_{\text{ln}} \), both with units \([\text{J/m}^2]\). Other components like fluxes of latent and sensible heat at the snow-air interface are relatively small and can be neglected.

The net average short-wave radiation \( R_{\text{sn}} \) expressed in \([\text{J/m}^2]\) on a certain day of the year, given by

\[
R_{\text{sn}} = R_{\text{ex}} \cdot (c_{\text{atm}} \cdot (1 - c_{\text{clo}} \cdot CC) + c_{\text{cor}}) \cdot c_{\text{veg}} \cdot c_{\text{alb}} \tag{4.5}
\]

Numerous earlier publications describe the different coefficients comprised in Equation 4.5 including the (for a detailed description of coefficients please refer to the notation section): attenuation of incoming short wave radiation by the atmosphere \( (c_{\text{atm}} \cdot c_{\text{clo}} \cdot c_{\text{cor}}) \) (Sauberer and Dirmhirn 1958), its transmission through the vegetation cover \( c_{\text{veg}} \) (Miller 1959) and its reflection at the snow surface \( c_{\text{alb}} \) (U.S. Army 1956). The mean seasonal cloud coverage \( CC \) in the region is derived from long-term meteorological records. The extraterrestrial radiation on a horizontal surface at a given location and time can be determined according to Milankovitch (1930). In 1957, Okanoue extended Milankovitch’s formulations for the computation of extraterrestrial radiation \( R_{\text{ex}} \) on a sloping surface, taking into account the inclination and orientation of slopes.

The total mean long-wave net radiation \( R_{\text{ln}} \) absorbed by the snowpack on a certain day of the year in \([\text{J/m}^2]\) consists of two components: (1) long-wave exchange between the air and the surface of the snowpack and (2) long-wave exchange between the vegetation canopy and the surface of the snowpack (Leavesley 1983, 1996):

\[
R_{\text{ln}} = (1 - f_c) \cdot \left( \varepsilon_{\text{air}} \cdot R_{\text{air}} - R_{\text{snow}} \right) + f_c \cdot \left( R_{\text{ai}r} - R_{\text{snow}} \right) \tag{4.6}
\]

The average long-wave heat of radiation emitted on melt days from (1) a perfect black body with an air temperature of \( R_{\text{air}} \) and (2) the snowpack (perfect black body) with a snow surface temperature of approximately 0 °C (273.16 K) \( R_{\text{snow}} \) can be estimated by applying the *Stephan Boltzmann Law*. It is assumed that the snowpack and the vegetation canopy behave as perfect black bodies \( (\varepsilon = 1.0) \), while the emissivity of air \( \varepsilon_{\text{air}} \) is a function of its moisture content based on meteorological observations. The forest cover density \( f_c \), which forms the predominant vegetation above the snowpack, can be derived from satellite images or vegetation maps.

Long-term meteorological data required for estimating spatially and temporally distributed melt factors include: 1) average cloud coverage in Equation 4.5, 2) average emissivity of air in Equation 4.6 and 3) average air temperatures used to estimate \( R_{\text{air}} \) in Equation 4.6 as well as to compute the melt factor in Equation 4.4.
Soil Moisture Store

The water balance of the soil moisture store \( S_t \) is based on the following equation:

\[
S_t(t+1) = S_t(t) + P_t(t)\Delta t + Q_{N}(t)\Delta t - E_{a}(t)\Delta t - Q_{c}(t)\Delta t
\]  

(4.7)

The soil layer may receive water directly though precipitation \( P_t \) (rain) and additional snowmelt water \( Q_N \) from the snow module.

The daily actual value of evapotranspiration \( E_a \) is computed on the basis of Thornthwaite's potential evapotranspiration \( E_p \) (Thornthwaite 1948), potentially limited by factors depending on 1) the daily level of soil moisture storage, 2) the mean daily precipitation, assuming the atmosphere approaches its saturated state when the rainfall rate exceeds any potential evapotranspiration, and 3) the spatial extension of snow cover, taking into account that evapotranspiration from a snowpack is very small and can be neglected. The Thornthwaite equation applies time series of air temperature, which is considered to be a good surrogate variable for estimating the potential evapotranspiration.

In the model the upper soil layer operates as an interface between the atmosphere and the ground (Figure 4.1a). It is the source of water losses through evapotranspiration into the atmosphere and allocates water exceeding the water holding capacity of the upper soil layer for runoff \( Q_c \) in a cascade consisting of three control stores for surface and sub-surface runoff. The first control store simulates the surface runoff, the second mimics the highly permeable soil layer processes which forms the source of interflow, and the third acts as the source for baseflow, being the deep zone aquifer of the soil profile.

The control stores are based on the simple bucket concept (Nash 1957, Nachtnebel et al. 1993) simulating a lateral flow component \( Q_l \) and the percolation of water \( Q_p \) to the next store in the cascade (Figure 4.1b).

Equation 4.8 and Equation 4.9 show that both flow components are calculated as linear functions of the storage height and the characteristic response times \( t_l \) and \( t_p \) for lateral flow and percolation, respectively. The lateral flow component \( Q_l \) stands for surface runoff \( Q_{sr} \), interflow \( Q_{in} \) or baseflow \( Q_{bf} \) and the water percolation component \( Q_p \) represents the vertical flow from the control stores for surface runoff to interflow \( Q_{sr-in} \) or flow from the control stores for interflow to baseflow \( Q_{in-bf} \). The catchment response time for lateral flow \( t_l \) is either \( t_{c,sr} \) (surface runoff), \( t_{c,fn} \) (interflow) or \( t_{c,bf} \) (baseflow) and for percolation \( t_p \) is represented by either \( t_{c,sr-in} \) (percolation from surface runoff to interflow control store) or \( t_{c,in-bf} \) (percolation from interflow to baseflow control store).

\[
Q_l = S \cdot \left(1 - e^{-\left(\frac{\Delta t}{t_l}\right)}\right) \frac{1}{\Delta t} \quad (4.8)
\]

\[
Q_p^{out} = S \cdot \left(1 - e^{-\left(\frac{\Delta t}{t_p}\right)}\right) \frac{1}{\Delta t} \quad (4.9)
\]

The outlet for lateral flow is lifted by \( h \), as a consequence water percolation to the deeper soil section lasting slightly longer than lateral flow at times of very low storage levels.

The water balance of the control stores for surface runoff, interflow and baseflow is based on the following three equations:
Surface Runoff Store

\[ S_{sr}(t+1) = S_{sr}(t) + Q_{sr}(t)\Delta t - Q_{sr}^{in}(t)\Delta t - Q_{sr}(t)\Delta t \]  \hfill (4.10)

The first bucket of the storage cascade receives the total water volume which is allocated for runoff. It is updated daily with short delay times for both the surface runoff \( Q_{sr} \) and the percolating water fraction to the interflow control store \( Q_{sr}^{in} \).

Interflow Store

\[ S_{in}(t+1) = S_{in}(t) + Q_{sr}^{in}(t)\Delta t - Q_{in}^{bf}(t)\Delta t - Q_{in}(t)\Delta t \]  \hfill (4.11)

\( Q_{sr}^{in} \) enters the interflow control store which releases water to the ground water \( Q_{in}^{bf} \) and produces interflow \( Q_{in} \).

Baseflow Store

\[ S_{bf}(t+1) = S_{bf}(t) + Q_{in}^{bf}(t)\Delta t - Q_{bf}(t)\Delta t \]  \hfill (4.12)

The last store of the cascade imitates the functioning of an aquifer so that it receives \( Q_{in}^{bf} \) as input and produces baseflow \( Q_{bf} \) as the only outflow component.

4.2.4 Climate Model Input

Many rainfall-runoff models use consistent yet potentially inaccurate estimates of climate inputs to replicate the gauged discharge at the closure section, assuming negligible significance as long as the model structure, and flexibility in parameter optimisation, are adequate. However, for the estimation of water balance the reproduction of the observed hydrograph alone is not satisfactory, especially for both spatially and temporally distributed water balance.

Spatial Estimation of Precipitation and Temperature

The aim of spatial prediction is to extrapolate from point information to spatially continuous data. Disaggregation of the catchment into sub-units requires hydro-meteorological input data over the area to be estimated from point observations for each sub-unit.

In order to achieve a good match between the spatial prediction surface and the local variability of the estimator, the reliability of any prediction technique is dependent on:

- The density of sample points and thus the observable variations in space and time.
- Prediction uncertainties, which are highest in high elevation zones due to a relative scarcity of monitoring sites.
- The spatial representativity of point observations for the relevant sites, a limitation in the case of a single climate monitoring station within a certain region that records only very local characteristics. Readings taken can deviate considerably from the mean characteristics of the region, only represented by that monitoring site.

Two contrasting approaches for statistical spatial prediction include (1) interpolation techniques and (2) empirical response models:

Dilemma of Interpolation Techniques
Interpolation models which assume distance as the only measure for similarity or dissimilarity are often inadequate for predicting climatic properties in high mountain regions, as the spatial variations caused by local influences are ignored.

Techniques like Ordinary Kriging and Universal Kriging (Deutsch and Journel 1992) do not define the intensity of variation on the basis of vicinity only, and remain stable with different sampling procedures. Details on interpolation methods applied for spatial predictions are given in Burrough (1985).

Spatial Reasoning through Response Models
Spatial reasoning modelling treats the observed variability of parameters (in this context a parameter is meant to be the estimator, either precipitation or temperature) as a response to other characteristic explanatory parameters. Values are thus deduced from the explanatory parameters for implementation as values for the predicted parameter, e.g. precipitation (Skoda 1993b, Chorley and Hagget 1965, Unwin 1969).

PRISM (Daly et al. 1994) exemplifies an advanced spatial prediction method based on a multiple regression approach, integrating a set of explanatory variables, the main one of which is topographic elevation. This approach was applied to generate the spatial distribution of mean annual precipitation for the European Alps (Schwarb et al. 2001). Lorenz and Skoda (Lorenz and Skoda 1997, 1998, 1999) published several benchmark studies on the spatial estimation of precipitation from point measurements in Austria. Loibl (1998) applied artificial neural networks to quantify multiple influences like orographic rising, rain shadow effects and large scale gradients of annual and seasonal spatial distributions, for precipitation in the Austrian Alps.

In general, multiple response models are less efficient when their predictions are applied to small time-scales e.g. daily precipitation. This is because the spatial interdependence of most explanatory variables is partially hidden behind the stochasticity of single events, such as convective rain showers.

Combination of Global Trends and Local Variations
3D-Kriging approaches combine assumptions of spatial correlation and the response concept (Cressie 1993). The External Drift Kriging procedure (Ahmed and De Marsily 1987, Bardossy 1990, Bardossy 1993), a Kriging procedure for non-stationary random functions, additionally allows good spatial predictions of climatic features in complex terrain, and small (daily) time scales. The estimator is either mean daily precipitation or temperature and the topographic elevation is preferably used as the surrogate variable, linearly correlated to the estimator. This method was applied to generate the spatial distribution of daily precipitation and temperature, summarised in terms of lumped sub-regional estimates.

4.3 Alpine Case Study Catchment

The transboundary alpine Gail catchment, selected as the case study, stretches almost linearly from West to East along the Italian - Austrian border line (12.7°-13.8°E and 46.4°-46.7° N) with the outlet at Federaun in Austria (Figure 4.3) and draining an area of 1305 km². The topographic elevation ranges from 600 m a.s.l on the valley floor to 2800 m a.s.l on the mountain ridges. The lower elevation zones are grasslands or arable lands devoid of forested areas, whereas the adjacent mountain slopes are typically covered by coniferous forests.
Figure 4.3: River network, topography, climate monitoring stations and location of the Gail catchment.

The Gail catchment is characterised by an alpine climate, dependent on Atlantic and Adriatic circulation patterns. The meso-climate within the catchment is highly variable due to the barrier effects of mountain ridges and the strong changes in elevation of the valley - ridge system. The mean annual precipitation lies around 1600 mm, the mean annual evapotranspiration is estimated to be around 530 mm and the mean annual temperature is around 6 °C with mean monthly temperatures varying from +12 °C in August to –1.5 °C in January.

Snow accumulation in winter and snowmelt in spring introduce a residual storage and lag to the hydrologic system. These delays cause a reduction in winter discharge, which is mainly fed by gradually depleting ground water. In spring the melting snowpack increases the discharge to its maximum level and recharges the ground water table. The hydrological characteristics of the Gail catchment have been analysed in detail by Fuchs et al. (1998) and Eder et al. (2001).

4.4 Parameter and Input Variable Estimation

This section focuses on input variable estimation and a priori parameterisation of dominant processes for the simulation of the seasonal water balance. It is the intention of the authors to demonstrate that the majority of parameter values necessary for a conceptual water balance model similar to the type selected, can be determined a priori without calibration. Nevertheless, some parameters do remain whose optimal values are difficult to estimate without calibration. The aim is to estimate parameter and variable values at the sub-regional scale, though some could only be defined at the sub-catchment scale. Water balance components are determined where possible for the sub-regional scale. In the following section spatial disaggregation of the Gail catchment is discussed prior to parameterisation and input variable estimation.
4.4.1 Spatial Disaggregation of the Gail Catchment

Five discharge gauging stations at Maria Luggau, Mauthen, Rattendorf, Nötsch and Federaun respectively delineate the sub-catchments of the Gail river whose tributaries reach from the head waters down to the main valley (Figure 4.4). The sub-catchments range in size between 144 km$^2$ and 368 km$^2$. Sub-catchments which stretch between two streamflow-gauging stations are named after the gauging station on the downstream closure section.

The orientation of the mountain ranges in relation to the Adriatic weather fronts have a strong influence on the climatic state of the Gail catchment. Precipitation levels and particularly the duration of the snowmelt period, differ significantly between the regions north and south of the Gail river. This is due to the barrier effects of the mountain range in the south (Karnischen and Julischen Alps) catching the precipitation before the range in the north (Gailtaler Alps). The difference in the aspects between the south and north facing slopes explains the snowmelt case. Thus each sub-catchment is subdivided into a northern and a southern section.

Furthermore sub-catchments are partitioned into four elevation zones, (1) valley sections up to 900 m a.s.l., (2) densely forested mountain slopes from 900 m a.s.l. to 1300 m a.s.l., (3) sub-alpine zone with Alpine woodland and mountain pastures from 1300 m a.s.l. up to 1700 m a.s.l., and (4) the high-alpine zone above the tree line around 1700 m a.s.l. This results in eight sub-regions per sub-catchment (with the exception of the sub-catchment Maria Luggau which has only six sub-regions due to the missing lowest elevation zone) and 38 sub-regions with a mean size of 35 km$^2$ for the total Gail catchment (Figure 4.4).

Figure 4.4: The setting of the 38 sub-regions within the Gail catchment.
4.4.2 Meteorological Model Input

The combined meteorological monitoring network of the Austrian and the Italian Hydrological Surveys consists of 39 precipitation and 15 temperature stations in the Austrian and Italian sections of the Gail catchment. The External Drift Kriging method was applied to generate the spatial distribution of daily precipitation and temperature. Precipitation is the only water balance component that is estimated on a fully distributed spatial scale. Distributions of long term mean precipitation for the summer and winter seasons are presented (Figure 4.5). Time series data of summarised daily precipitation and temperature in terms of 38 lumped sub-regional estimates between 1971 – 1995 served as model inputs for the water balance study.

Comparing long term mean regional values of Thornthwaite's potential evapotranspiration with estimates of actual evapotranspiration (difference between observed long term mean precipitation and discharge) shows that the Thornthwaite method underestimates evapotranspiration. According to Milly (1994) and based on own experiences through studies in the Alpine region, potential evapotranspiration derived from the Thornthwaite method was multiplied by a factor of 1.25.

4.4.3 Discharge form Sub-catchments

Daily measurements from automatic streamflow gauging stations are readily available for five locations, delineating the five sub-catchments of the Gail river. Streamflow is a spatial integrator for hydrological processes, thus the network of streamflow gauging stations (approx. 1 station/100 km²) provides highly valuable inputs for water balance simulations on the sub-catchment scale. In sequentially interconnected sub-catchments, where neighbouring sub-catchments located upstream drain into the sub-catchment of interest, then gauged streamflow is also applied as a model input (Figure 4.2 and 4.4). Runoff measurements are also required for estimation of streamflow recession parameters and the validation of the model results discussed in later sections.

Studies of neighbouring catchments have concluded that groundwater outflow from each sub-basin is below 1.5 % of total discharge (Eder et al. 2001). Consequently, it is assumed that potential uncertainties introduced by unrecognised subsurface flow at the closure section are not of significance.
4.4.4 Snowmelt and Accumulation

Data from three climate monitoring stations, Reisach (646 m a.s.l.), Kornat (1025 m a.s.l.) and Villacher Alpe (2135 m a.s.l.), representative of the entire Gail catchment, were analysed to define intervals of transitional air temperature. It was found that values for $T_{t,\text{min}}$ and $T_{t,\text{max}}$ were similar, in spite of the differing location and altitude of selected monitoring stations. Transitional air temperatures were determined at the lumped catchment scale and values of $T_{t,\text{min}}$ and $T_{t,\text{max}}$ set to -0.75 °C and 3.25 °C respectively. Similar parameter values for air temperature transitions were found by Braun (1985) for various catchments in Germany and Lauscher (1982) for the Vienna region.

The melting factor $m_f$ is estimated at the lumped sub-regional scale. Values for the 21st of March show remarkable differences in the 38 sub-regions (Figure 4.6a). Parameter values in the northern and highly elevated parts of the catchment are significantly higher than those of the southern and lower sections. This can be explained by higher short-wave radiation fluxes on south facing and highly elevated mountain slopes, as opposed to non-forested regions on the northward facing valleys. Contributions by long-wave radiation fluxes, arising for instance from forested areas on mid-elevation zones in the Gail catchment, seem less significant in the calculation of the melt factors.

4.4.5 Soil Properties

Soil properties have been mapped in detail for agricultural areas on the valley floor and on the lower mountain slopes (Österreichische Bodenkartierung 1960, 1964). For the remaining regions the soil map of the Austrian
Academy of Sciences (Österreichische Akademie der Wissenschaften 1979) is available. Estimates of the local soil properties are comparable with typical values found in Dingman (1994).

Soil depth decreases gradually with increasing elevation and slope gradient. In the highest elevation zones, soil storage capacity is negligible, with bare rock being exposed to the surface in the vicinity of ridges. Soil properties are collected for all sub-regions of the Gail catchment. Rendzina, with 30% spatial coverage is the predominant soil type especially in the elevated zones, followed by Cambisol (22%), Podsol (21%), Lithosol (17%), Fluvisol (8%) and Ranker (1%); approximately 1% of the study area shows bare rock.

![Figure 4.6: Model parameter: distribution (a) of the soil depth ($D_{sp}$ in [mm]) and (b) of the melt factor ($mf$ in [mm/K/day], date: 21st of March): Estimates for the 38 sub-regions of the Gail catchment.](image)

The permanent wilting point $\theta_{pw}$ and the field capacity $\theta_{fc}$ are calculated according to the step-wise methodology of Baumer (1989) for all profile layers of each soil type. This methodology utilises available information on fractions of gravel and rocks, sand, loam, clay, humus as well as the calcium content and the pH value. Estimates of the permanent wilting point, field capacity and total soil profile depth to an impervious layer $D_{sp}$ allow definition of total soil moisture capacity of the soil profile above the permanent wilting point $C_{wp}$ and the soil moisture capacity up to the field capacity of the soil profile $C_{fc}$ (Table 4.1 and Figure 4.6b).
Table 4.1: Mean lumped estimates of soil properties for 38 sub-regions of the Gail catchment. The codes of the sub-regions are assigned in brackets and grouped into sub-catchments.

<table>
<thead>
<tr>
<th>Maria Luggau</th>
<th>Mauthen</th>
<th>Rattendorf</th>
<th>Nötsch</th>
<th>Federaun</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_p$ [mm]</td>
<td>$\theta_{pwp}$ [I]</td>
<td>$\theta_{fc}$ [I]</td>
<td>$D_p$ [mm]</td>
<td>$\theta_{pwp}$ [I]</td>
</tr>
<tr>
<td>400</td>
<td>0.04</td>
<td>0.14(1)</td>
<td>320</td>
<td>0.03</td>
</tr>
<tr>
<td>660</td>
<td>0.07</td>
<td>0.19(2)</td>
<td>700</td>
<td>0.07</td>
</tr>
<tr>
<td>760</td>
<td>0.08</td>
<td>0.24(3)</td>
<td>580</td>
<td>0.06</td>
</tr>
<tr>
<td>638</td>
<td>0.06</td>
<td>0.17(4)</td>
<td>700</td>
<td>0.07</td>
</tr>
<tr>
<td>800</td>
<td>0.08</td>
<td>0.25(5)</td>
<td>800</td>
<td>0.08</td>
</tr>
<tr>
<td>800</td>
<td>0.08</td>
<td>0.25(6)</td>
<td>790</td>
<td>0.08</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>800</td>
<td>0.08</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>800</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Depth of the soil profile $D_p$, permanent wilting point $\theta_{pwp}$, and field capacity $\theta_{fc}$.

4.4.6 Catchment Drainage

The flow rates of surface runoff $Q_{sr}$, interflow $Q_{in}$ and baseflow $Q_{bf}$ are conceptualised as linear storage functions with characteristic delay times being constants given by each of $t_{c, sr}$, $t_{c, in}$ and $t_{c, bf}$.

While theory relates the characteristic travel time of subsurface flows to regional characteristics, including such variables as the saturated hydraulic conductivity, soil depth and porosity (Brutsaert and Nieber 1977), travel times are in practice estimated directly from the recession curves, derived from sub-catchment discharge measurements. Mountain basins do not adequately enable response time estimation for sub-regional runoff components based on the above theory. Attempts to use such theoretical methods neither have the capacity to estimate hydraulic conductivity nor the flow characteristics, which are unknown for fractured rock systems.

In this study, delay time parameters are estimated a priori through an inverse procedure of extracting recession curves from runoff measurements in sub-catchments (Wittenberg 1994). Individual recession curves separately covering a large range of discharge flows can be merged into a single recession curve - called master recession curve - in the absence of intervening precipitation and snowmelt events. Linear recession functions are matched to the master recession curve.

At the sub-catchment and larger scales there is a gradual transition between the merged flows of surface runoff, interflow and baseflow. Baseflow response time for alpine catchments, can be determined from the lowest part of the master recession curve for winter low flow, since winter season discharge results mainly from ground water drainage. Delay time for interflow is estimated from the highest section of the main recession curve. The non-linear transition section in the log-plot is omitted from the analysis. Delay time for surface runoff is estimated on the basis of the digital river network, distance of flow path to the closure section and the mean flow velocity.

Factors delaying water percolation, $t_{c, sr-in}$ and $t_{c, in-bf}$, from surface runoff to interflow control stores, and from interflow to baseflow, respectively, are difficult to estimate a priori. For this reason, parameters were
calibrated by visually optimising the best fit curves for observed and simulated monthly discharge. Estimated response times for all runoff components are presented (Table 4.2) for the five selected sub-catchments of the Gail river. Correlation of drainage parameters with surrogate variables, such as sub-catchment size and mean topographic elevation (Table 4.2) is significant at this scale. Nevertheless, extrapolations of drainage parameter values from the sub-catchment scale to the sub-regional scale based on those relationships are avoided because of the parameter’s sensitive to model performance with decreasing spatial scale, especially in mountainous catchments.

Table 4.2: Catchment drainage characteristics for the Gail sub-catchments. The mean topographic elevation and the physical size of the sub-catchments are listed.

<table>
<thead>
<tr>
<th>Sub-Catchment</th>
<th>Maria Luggau</th>
<th>Mauthen</th>
<th>Rattendorf</th>
<th>Nötsch</th>
<th>Federaun</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{c,sr}$</td>
<td>0.8</td>
<td>0.9</td>
<td>0.9</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$t_{c,sr-in}$</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$t_{c,in}$</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$t_{c,in-bf}$</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>$t_{c,bf}$</td>
<td>115</td>
<td>120</td>
<td>125</td>
<td>160</td>
<td>170</td>
</tr>
<tr>
<td>Elev. [m a.s.l.]</td>
<td>1835</td>
<td>1579</td>
<td>1263</td>
<td>1073</td>
<td>1098</td>
</tr>
<tr>
<td>Size [km²]</td>
<td>144</td>
<td>203</td>
<td>249</td>
<td>341</td>
<td>368</td>
</tr>
</tbody>
</table>

Catchment response times of surface runoff $t_{c,sr}$, interflow $t_{c,in}$, baseflow $t_{c,bf}$ and percolation flow components from surface runoff to interflow control stores $t_{c,sr-in}$, and from interflow to baseflow control stores $t_{c,in-bf}$.

### 4.5 Interpretation and Evaluation of Results

The results of the semi-distributed water balance model concept, applied to the Gail catchment, are presented at various spatial scales.

#### 4.5.1 Water Balance at Sub-Regional Scale

The accumulation and depletion of snow as well as soil moisture fluctuations are modelled on the sub-regional scale.

In the highest sub-regions, maximum snowpack water equivalent occurs in April. In contrast to the high alpine zones with around 500 mm calculated mean monthly sub-regional snowpack water equivalent, the lowland regions of the Gail catchment are nearly snow free by April (Figure 4.7a). The mean monthly fluctuations of lumped sub-regional snowpack water equivalents, e.g. in the sub-regions number 1 and number 2 (compare with Figure 4.4) of the sub-catchment Maria Luggau and sub-regions no. 37 and no. 38 of sub-catchment Federaun are presented in Figure 4.8a.

Notably, snowpack depth is not a linear function of stored water in the snowcover because of the countless processes taking place in the snowpack, including the settling of snow as a consequence of snow weight and alternative melting as well as re-freezing. Temperature fluctuations, distribution of stored snowpack water, as
well as the variability of the melt factor over time and space determines snowmelt runoff from the 38 sub-regions.

Figure 4.7: Model output: mean monthly storage estimates (a) of snowpack water equivalent \( (S_W) \) and (b) of soil moisture \( (S) \) in the 38 sub-regions for the 1971-1995 period.

The time averaged spatial variability in soil moisture for May over the analysis period, in terms of lumped sub-regional estimates, is presented in Figure 4.7b whereas Figure 4.8b shows the intra-annual variation of soil moisture storage in four selected sub-regions. Compared to water storage values in the snowpack, soil moisture is highest in May and varies less over space and time because the soil acts as a smoothing reservoir to the inputs of snowmelt and precipitation:

- Soil moisture increase after heavy precipitation events is constrained by the low water holding capacities of soils in high alpine regions.
- Generally, soil moisture storage does not significantly decrease in summer due to the moderate climate and short intervals between storms, especially in elevated zones.
- In winter, soil drainage is impeded by soil frost.

Actual evapotranspiration is also modelled on the sub-regional scale, as a function of potential evapotranspiration, spatial snow cover fraction, precipitation and soil moisture.
4.5.2 Water Balance at Sub-Catchment Scale

The monthly lumped sub-catchment values of surface runoff, interflow and baseflow are calculated for all sub-catchments. Other water balance components at the sub-catchment scale, such as evapotranspiration and precipitation, have been summarised from smaller scale spatial estimates. Figure 4.9 shows the intra-annual variations of snowmelt runoff, surface runoff, interflow and baseflow for the sub-catchment Rattendorf. These results were also calculated for all sub-regions of the remaining sub-catchments. Figure 4.10 presents the long term monthly results of discharge as well as actual evapotranspiration and precipitation for the total Gail catchment as the mean of all sub-catchment results. Variations of sub-catchment results within the total Gail catchments are indicated by interval 'bars'.
Figure 4.9: Long term mean inter-annual variation of the snowmelt ($\langle \bar{Q}_N \rangle$) and the runoff components: surface runoff ($\langle \bar{Q}_{sr} \rangle$), interflow ($\langle \bar{Q}_{in} \rangle$) and baseflow ($\langle \bar{Q}_{bf} \rangle$) for the period 1971 - 1995 in the Gail sub-catchment Rattendorf.

Figure 4.10: Long term mean inter-annual variation of precipitation ($\langle \bar{P}_s \rangle + \langle \bar{P}_r \rangle$), actual evapotranspiration ($\langle \bar{E}_a \rangle$) and discharge ($\langle \bar{Q}_{sr} \rangle + \langle \bar{Q}_{in} \rangle + \langle \bar{Q}_{bf} \rangle$) for the period 1971 - 1995 in the total Gail catchment until the closure of Federaun and five sub-catchments: printed intervals indicate the variation of the water balance among the Gail sub-catchments Maria Luggau, Mauthen, Rattendorf, Nötsch and Federaun.
4.5.3 Evaluation of Model Results

Modelled spatial and temporal distributions of precipitation, temperature and associated potential and actual evapotranspiration in the Gail catchment are similar to studies carried out for the same catchment (Fuchs 1998), for the Austrian region (Loibl 1998), and for the entire Alps (Baumgartner et al. 1983; Schwarb et al. 2001).

The performance of the water balance model is evaluated by comparison of simulated and observed monthly discharges at five streamflow gauging station locations. Figure 4.11a lists the long-term mean monthly discharge, the maximum and minimum discharge, the coefficient of correlation and the coefficient of variation for monthly discharge for all sub-basins between 1971-1995.

<table>
<thead>
<tr>
<th>Sub-catchment</th>
<th>Mean [mm]</th>
<th>Max [mm]</th>
<th>Min [mm]</th>
<th>r</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maria Luggau</td>
<td>77</td>
<td>353</td>
<td>17</td>
<td>0.95</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>78</td>
<td>363</td>
<td>9</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td>Mauthen</td>
<td>79</td>
<td>363</td>
<td>12</td>
<td>0.98</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>79</td>
<td>349</td>
<td>14</td>
<td></td>
<td>0.79</td>
</tr>
<tr>
<td>Rattendorf</td>
<td>80</td>
<td>370</td>
<td>14</td>
<td>0.99</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>77</td>
<td>376</td>
<td>13</td>
<td></td>
<td>0.74</td>
</tr>
<tr>
<td>Nötsch</td>
<td>81</td>
<td>365</td>
<td>19</td>
<td>0.98</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>81</td>
<td>315</td>
<td>20</td>
<td></td>
<td>0.65</td>
</tr>
<tr>
<td>Federaun</td>
<td>86</td>
<td>470</td>
<td>23</td>
<td>0.99</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>86</td>
<td>350</td>
<td>23</td>
<td></td>
<td>0.60</td>
</tr>
</tbody>
</table>

Observed ($Q_o$) and predicted ($Q_p$) long term mean monthly (mean), maximal monthly (max) and minimal monthly discharge (min), the coefficient of correlation ($r$) and the coefficient of variation ($CV$).

Figure 4.11: Performance of the semi-distributed hydrological model: (a) Accuracy criteria for all Gail sub-catchments over the 1971-1995 period; (b)&(c) histogram giving the difference of the observed and the predicted monthly discharge at closure sections Maria Luggau and Federaun; (d)&(e) comparison of observed and predicted long term mean monthly discharge at gauging locations Maria Luggau and Federaun.
Taking into account that model parameters are *a priori* estimates obtained without any use of an automatic calibration tool, the performance of the hydrological model is very good, with the coefficient of correlation ($r$) always remaining above 0.95. One specific simulated discharge in October 1993 which is significantly lower than the observed, is the maximum value at the closure section at Federaun. A detailed analysis of the problematic situation with an under-estimation of discharge could not be fully resolved to date, leaving a number of uncertainties. Without any clear evidence, an alteration of model inputs or model parameter values is avoided.

Figures 4.11b and 4.11c show frequency plots for differences in observed and predicted monthly discharge from sub-catchments Maria Luggau and Federaun, respectively furthest up and downstream. Systematic prediction errors can be ruled out from the regular normal distribution of the differences. Figures 4.11d and 4.11e present a good correspondence between predicted and observed flow regimes at Maria Luggau and Federaun. The same standard of performance was obtained for the remaining three sub-catchments, not presented for the sake of brevity.

### 4.6 Summary, Conclusions and Outlook

A methodology for estimating the semi-distributed water balance of alpine catchments on monthly and spatial lines was developed, and includes:

- A parsimonious water balance model based on dominant hydrological processes typical for an alpine catchment
- The definition of an appropriate level of catchment disaggregation, allowing effective estimation of distributed model parameters and input data
- The modelling of the spatial distribution of meteorological data
- Approaches for *a priori* estimation of model parameters

The concept was applied to the Gail catchment located in the Austrian-Italian Alps. Results show that the water balance concept is highly feasible for the Gail catchment. Selected as a representative alpine catchment, it is anticipated that methods for semi-distributed seasonal water balance estimation in the Gail catchment will be applicable to other alpine regions.

The model complexity applied to the Gail catchment is minimised and incorporates only a small set of model parameters. For other alpine catchments, model complexity might be reduced by condensing three runoff modules into two, the first for simulating saturation excess runoff and the second for modelling sub-surface runoff. Furthermore, parsimonious water balance models require only the estimation of limited parameters as a prerequisite for *a priori* model parameterisation.

Distributed water balance computation is constrained by the spatial scale of parameter and input data. Snowpack accumulation, snowmelt runoff, evapotranspiration dynamics and fluxes in the upper soil layer were modelled on the sub-regional scale. Surface and subsurface runoff generation were parameterised on the sub-catchment scale, hence surface runoff, interflow and baseflow estimates are limited to that scale. A minimum appropriate scale was used for spatial resolution of the results data, where this level would apply to
any modelling procedure using the same data availability to model monthly water balance in a high mountain region.

Recession analysis showed that catchment response times do not change significantly with varying durations of selected and analysed discharge periods. Indeed, analysed discharge periods can be very short without contributing any significant uncertainty to the recession parameters, providing that the few selected recession curves cover the full range of flow variability. In ungauged catchments it is often feasible to measure the discharge at the respective outlets over a short period, i.e. during low flow in winter and high flow in spring, to obtain correct catchment response times. In doing so, the number of sub-catchments could be increased, thereby reducing the spatial scale for estimated runoff components and decreasing the potentially distotionary gap between the sub-regional and the sub-catchment scale.
Chapter 5:  
Water Balance Modelling with Monthly Hydro-Meteorological Input

**Keywords:**  
- monthly water balance  
- hydrologic modelling  
- monthly climate input  
- lumped catchment scale  
- statistical disaggregation in space and time  
- Gail catchment

### 5.1 Introduction

Monthly water balance models sometimes perform insufficient in alpine regions with boreal climate conditions. The problems arise when attempting to model the monthly water balance with lumped mean monthly climate data. Processes which take place at smaller spatial scales than the catchment scale and smaller temporal scales than the monthly scale are important determinants of the monthly water balance. For the Upper Enns catchment it was found that the temporal distribution of monthly mean temperature was enough to reach good model performance (Chapter 2.6.3). For many other alpine catchments with different meteorological characteristics this concept may not be fully applicable. This is particularly true for the Gail catchment which is influenced by continental, Mediterranean and Adriatic weather systems. Therefore, the Gail catchment up to the closure at Federaun is selected as the case study catchment. The meteorological characteristics as well as the physiographic properties of the Gail catchment were described in Chapter 4.

The basic structure of the monthly water balance model was developed in Chapter 2.6.1 This includes the soil moisture accounting module formulated as a so called Manabe bucket (Figure 2.9, Manabe 1969) which is combined with a snow module that applies the temperature-index algorithm (Equation 2.16). The experiences gained in Chapter 2 as well as the results in Chapter 3 (Table 3.3) lead to the conclusion that, when focusing on the soil moisture accounting module only, a parsimonious structure with six to eight model parameters provides sufficient detail to simulate dominant hydrological processes of the monthly water balance.

**Scientific Objectives**

It would be more convenient, however, to use a deterministic, continuous hydrological model using physical model parameters and monthly input data for large scale hydro-meteorological applications. The models developed in this chapter statistically disaggregate catchment mean monthly climate data into smaller spatial scales and time scales and reach an acceptable level of performance. Statistical disaggregation concepts can be parameterised on the basis of analysing available data from climate monitoring stations distributed throughout catchments.

Hence, the scientific objectives of this research are:

- To study and identify the characteristic meteorological features that emerge at smaller scales than the catchment scale and the monthly time step.
To define the dominant scales and correlation of distributed temperature and precipitation in space and time which have the most impact on the monthly water balance. These results can then be used for conceptualising water balance models.

To develop monthly water balance models through hypothesis testing thereby evaluating the model performance with a special focus on the number of applied model parameters.

Outline of the Chapter
First, a detailed overview on the spatial and temporal distributions of precipitation and temperature is provided and tables as well as figures of the results are copied to Appendix 2 for the sake of better readability. Data analysis is based on daily time series of climate monitoring data which were available from 15 temperature and 39 precipitation stations for the period between 1971-1995. Hydro-meteorological and physiographic properties may in various degrees influence the annual and monthly water balance of alpine catchments.

Second, various water balance models which apply different concepts of spatial and temporal disaggregation of monthly climate input data are formulated using a step by step development procedure based on performance analysis and hypothesis testing. All models built on parsimonious model structures only account for the dominant hydrological processes that emerge, allow for stepwise model conceptualisation, and apply the least possible number of model parameters. The performance of models is documented through annual and monthly signature plots and hydrographs as well as through direct performance measures.

Third, model development results in recommendations on how to set up water balance models which apply monthly climate data to simulate monthly water balances in alpine catchments.

5.2 Distribution of Precipitation and Temperature in Space and Time

5.2.1 Elevation Distribution
Elevation is analysed first because it is often correlated with the spatial distribution of climate variables. Topography is not a time-dependent property, however, its influence on precipitation and temperature is variable over the annual cycle depending on weather circulation patterns. Figure A2.1 presents the histogram of the spatial distribution of elevation \( \text{frq}(H) \) within the Gail catchment. Table A2.1 lists the catchment fractions of nine elevation zones between the lowest elevation sections of around 600 m a.s.l. and the highest peaks at 2800 m a.s.l.

5.2.2 Spatial and Temporal Distribution of Temperature

Next the regression of observed mean monthly temperature and elevation data \( \langle T_{st} \rangle = a \cdot H + b \) gathered on the monitoring stations is examined and results are presented in Figure A2.2. Figure A2.2 as well as the following figures show results for six months, January, March, Mai, July, September, and November and provide a good overview of the climate properties analysed. The correlation between elevation and temperature is strongest in summer and autumn (July and September) and also fairly strong in spring and early winter. In the winter months, the characteristic decrease of temperature with increasing elevation is still captured by the fitted linear regression
the correlation becomes much weaker (compare with the January-plot in Figure A2.2). The spatial distribution of temperature within the catchment is analysed with mean monthly temperature data from monitoring stations \((T_{air})\) distributed throughout the Gail catchment. The distribution of mean monthly temperature data is slightly skewed and therefore was transformed by applying Schröder’s transformation equation (Schröder 1969).

**Transformation (Schröder 1969):**

\[
H' = \left( \frac{H - H_{\text{min}}}{H_{\text{max}} - H_{\text{min}}} \right)^C
\]

\(H_{\text{min}} = 491 \text{ m a.s.l., } H_{\text{max}} = 2239 \text{ m a.s.l.}\)

\(C = 0.77; \quad CS = 0, \quad H' = 0.43, \quad \sigma_{stT} = 0.22\)

Literature regarding data transformation is plentiful (see Hartung 1999, for a list of key references; Atkinson 1973; Box and Cox 1964; Cunnane 1978; Schröder 1969). This study does not primarily focus on transformation of empirical data because accurate data sources and satisfactory sample sizes for statistical analyses from monitoring stations at elevations above 1500 m are difficult to obtain. In addition, estimating the regional distribution of climate variables is challenging because of the small number of stations in the climate monitoring network.

Results show that transformed mean monthly temperature data (see Schröder 1969) \((F(T_{air}))\) is distributed normally within the catchment and is plotted in Figure A2.3 \((F(T_{air}))\) together with the fitted analytical cumulative distribution function (cndf). The mean coefficients of variation \((CV_{T,space}^{1-12})\) are listed in Table A2.2 for all months. The temporal distribution of temperature \((F(T_{air})\)) was examined with catchment mean daily temperature \((T_{mean})\). Thus, the analysis of observed daily temperature data results in the spatio-temporal distribution of temperature \((F(T_{mean})\)\). Both, the temporal and the spatio-temporal distribution of the empirical data follow a normal distribution. Plotted empirical data (plotting position according to Blom 1958) and fitted analytical cndf are shown in Figure A2.4 and Figure A2.5. Estimated coefficients of variations \((CV_{T,space}^{1-12} \text{ and } CV_{T,space-time}^{1-12})\) are tabulated in Table A2.3 and A2.4.

Often the interdependencies of hydro-meteorological properties are important. Hence, for an accurate statistical description multivariate distributions may be more appropriate than univariate relationships. For example, if mean monthly temperature and precipitation are spatially correlated, it might be difficult to model the monthly water balance with monthly temperature and precipitation data without capturing the spatial correlation of both variables. The application of multivariate distributions is somewhat limited by the sample size. If for a univariate statistical analysis a sample size \(M\) is found to be necessary then \(M^2\) sample data should be available for a bivariate analysis and \(M^3\) for a trivariate analysis (Weingärtner 1979). For that reason multivariate statistical analyses with more than two variables often lack enough empirical data. It is possible to combine univariate and bivariate methods in order to include more than two variables. The bivariate normal probability distribution (bnpdf) is well described in the literature (Johnson and Kotz 1972; Fahrmeier et al. 1996). Sackl (1994) applies bivariate statistical methods in case studies on generating design flood hydrographs.
and precipitation characteristics. Descriptions of several other types of multivariate distributions may also be found in the literature (e.g. Raynal-Villasenor and Salas 1987).

In the following empirical data on temperature, elevation and precipitation are analysed with a bivariate statistical method on the basis of bnpdf. The marginal distributions of bnpdf are univariate npdf. Hence, this concept is best suitable because univariate distributions of temperature in space and time are normal distributed. Distributions of precipitation and the distribution of elevation can be transformed into normal distributions applying a log-transformation of empirical precipitation data and eliminating the skewness in the empirical elevation sample (Equation 5.1), respectively. This concept of applying a transformation to the empirical raw data is applied in the current work. Instead, another possibility is, first, to fit any appropriate analytical pdf other than a npdf. Second, the gained pdf, e.g. of the log-normal or gamma type, is transformed into a npdf (Sackl 1994).

The spatial distribution of temperature may be expressed also as bivariate distribution of temperature and elevation. The transformed elevation ($H'$, $C_{st_H} = 0$, Figure A2.6) and the mean monthly temperature ($\bar{T}_{st}$) are well correlated for the March-October period (Table A2.5). Significant lower coefficients of correlation ($r_{T_{st}-H_{space}}^{1-12}$) are found in winter from November to January. This situation is also well shown in Figure A2.7a which presents the bivariate plot of empirical mean monthly temperature ($\bar{T}_{st}$) and transformed elevation ($H'$) data. The data is more scattered in January and November than this is the case for March to September, especially for May and July. The elliptic iso-lines ($\alpha = 0.99, 0.9, 0.5$ and $0.1$) represent lines of equal probability estimates of the analytical bnpdf ($f(T_{st}, H', r)$). Figure A2.7b shows a three dimensional plot of the fitted and standardised bnpdf.

The correlation ($r_{T_{st}-H_{space-time}}^{1-12}$) between observed mean daily temperature from 15 temperature monitoring stations ($T_{st}$) and the transformed elevation of these monitoring stations ($H'$) is lower the in the case of mean monthly temperature (Table A2.6). In Figure A2.8 the bivariate distribution of daily temperature and elevation is presented ($f(T_{st}, H', r)$). Figure A2.8a shows that the empirical data is scattered which indicates a low correlation.

5.2.3 Spatial and Temporal Distribution of Precipitation

Here, the spatial and temporal distribution of precipitation is studied in the same way as this has been done for temperature above. The results are presented in the Figures A2.9, A2.10 and A2.12-A2.15 which may be interpreted similarly as done for temperature, hence, only special features of the distributions are discussed in the following.

The correlation between precipitation is stronger in the summer season than in winter (Figure A2.9). The increase of mean monthly precipitation ($\bar{T}_{st}$) per 1000 m elevation increase ranges between about 10 mm in spring (March) and about 35 mm in summer (July).
Log-transformed mean monthly precipitation ($\bar{P}_{st}$) is spatially normal distributed throughout the catchment (Figure A2.10). The long term mean monthly coefficients of variation ($CV_{P_{space}}^{st}$) of the empirical data ranges between the minimum in August (0.03) and the maximum in January (0.06) (Table A2.7).

Daily catchment precipitation ($P_{cmt}$) follows a Gamma-distribution within a month. The cumulative frequency of catchment daily precipitation estimates and the analytical cumulative Gamma-distribution function is presented in Figure A2.11 ($F(P_{cmt})$). The respective long term mean monthly values of coefficients of variation ($CV_{P_{time}}^{cmt}$) are listed in Table A2.8 and are quite evenly distributed throughout the year.

In Figure A2.12 the results of the distribution analysis of monitored daily precipitation ($P_{m}$) is shown. This is called the spatio-temporal distribution of precipitation ($F(stP)$) which maps the spatial distribution of point observations within the catchment and the spatial distribution of daily data within a month follows again a Gamma-distribution. Mean monthly values of the coefficients of variation ($CV_{P_{time}}^{cmt}$) are evenly distributed intra-annually (Table A2.9).

Next the bivariate statistical distributions of precipitation and elevation is studied. Table A2.10 and Table A2.11 present the coefficients of correlation of elevation and mean monthly ($r_{P_{time}H_{space}}^{st}$) as well as between elevation and daily precipitation ($r_{P_{time}H_{space}}^{cmt}$). Listed values describe a very weak correlation in both cases.

The scattered empirical data in Figure A2.13a ($\bar{P}_{st},H'$) and Figure A2.14a ($P_{cmt}',H'$) is an indication of low correlation between the spatial distribution of precipitation and elevation. Especially, Figure A2.14a captures the situation very well because daily precipitation values which are observed at a single monitoring station may vary within a large interval in all months. Hence, precipitation data points of a single climate station are lined up at the same elevation ($H'$).

### 5.2.4 Bivariate Distributions of Combined Precipitation and Temperature

The spatial, temporal and spatio-temporal correlation of temperature and precipitation is indicated in Table A2.12, Table A2.13 and Table A2.14, respectively. The coefficients of correlation between mean monthly point observations ($\bar{P}_{st}$ and $\bar{T}_{st}$) is highest ($r_{P_{time}T_{time}}^{st}$), whereas the correlation measures of daily mean catchment estimates ($P_{cmt}$ and $T_{cmt}$; $r_{P_{time}T_{time}}^{cmt}$) and daily point observations ($P_{st}$ and $T_{st}$; $r_{P_{time}T_{time}}^{st}$) within a month are significant lower.

The horizontally aligned elliptic iso-lines describing the bpdf (Figure A2.15a, Figure A2.16a and Figure A2.17a) are horizontally orientated which is an indication for low correlation measures. Plots of the analytical bpdf ($f(P_{cmt}',T_{cmt},r)$, Figure A2.15b; $f(P_{cmt}',T_{cmt},r)$, Figure A2.16b; $f(P_{cmt}',T_{cmt},r)$, Figure A2.17b) are characterised by low probabilities and a wide spreads.

In Johnson and Kotz (Johnson and Kotz 1972) the interpretation of multivariate statistical plots is well described upon a selection of examples.

The statistical analysis of climate data gives hints on which relationships may be more appropriate than others for the integration in an monthly water balance model concepts. However, e.g. low correlation coefficients
between mean monthly precipitation and temperature point observations does not necessarily mean that the bivariate distribution of monthly temperature and precipitation data is meaningless in an monthly water balance model. It might not be the climate forcing which is an emergent process, but the distribution of climate inputs which results in distributed snow depths may be a dominant property of the monthly water balance. Hence, testing of different concepts in order to evaluate pre-defined hypothesis is important.

In the next section the spatial and temporal distributed information is being applied and tested step by step in various versions of monthly water balance models. First it is started with a parsimonious model concept with lumped physiographic parameter estimates as well as lumped monthly values. This is then adjusted according to the deficits detected through evaluation on performance measures, analysis of graphical displayed results and hypothesis testing. After each modification step a new hypothesis is created. The model performance is evaluated by comparison of observed and modelled annual as well as monthly discharge. Step by step more and more disaggregated information in space and time is being incorporated in the model concept in order to improve the performance.

Such a semi-objective process of model development which includes explicitly the heuristic experience of the modeller helps to avoid unsound model structures. Moreover it results in parsimonious water balance models with the least amount of complexity that accounts only for the emergent properties of the monthly water balance.

Finally, several different modelling approaches were tested which are listed in the following section.

### 5.3 Development of Monthly Water Balance Models

#### 5.3.1 Model Concept with Lumped Monthly Climate Inputs

Model 1 (M1, in the same style models 2-23 are abbreviated with M2-M23) is a soil moisture accounting model which is based on the so called "Manabe bucket" (Chapter 2.4.1) and applies a temperature-index snow module (Figure 2.9). Only two model input variables and six model parameters are necessary for the operation of M1. These are monthly mean catchment precipitation ($P$) and temperature ($T$) data, mean catchment estimates of the snowmelt factor $mf$, transition air temperature $T_t$, threshold air temperature $T_0$, total profile storage capacity $C_{p}$, catchment response time for sub-surface flow $t_c$ and mean number of days with precipitation $T_{wet}$. Table 5.1 lists all model parameters, some of which have already been defined as distributed parameters in Chapter 4 and are now summarised in terms of lumped catchment values.

<table>
<thead>
<tr>
<th>model parameter</th>
<th>estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle T_{wet} \rangle$ [days]</td>
<td>16</td>
</tr>
<tr>
<td>$mf$ [mm/K/d]</td>
<td>1.50</td>
</tr>
<tr>
<td>$T_t$ [°C]</td>
<td>1.25</td>
</tr>
<tr>
<td>$T_0$ [°C]</td>
<td>0.0</td>
</tr>
<tr>
<td>$C_p$ [mm]</td>
<td>285</td>
</tr>
<tr>
<td>$t_c$ [mm]</td>
<td>47</td>
</tr>
</tbody>
</table>
The result derived with M1 is similar to that in Chapter 2 for the Upper Enns catchment. Thus, the annual water balance (Figure 5.1a and 5.1b) can be predicted well with a modelling concept of such great simplicity. However, the intra-annual water balance is not well reproduced (Figures 5.1c and 5.1d). Discharge in winter is under-predicted and much too high in late spring. The reason is that mean monthly temperature values stay below freezing over many months during the winter season. Therefore, the model does not account for short term temperature fluctuations reaching above the transition air temperature \( T_t \) (rainfall or snowfall) or the threshold air temperature \( T_0 \) (frost or melt), which means that all precipitation is modelled as snowfall, accumulates to a snow cover without any intermediate melting period; a situation which is not realistic. Similarly, the model simulates always snowmelt when positive mean monthly temperatures in spring do not reflect within month temperature fluctuation.

In Table 5.8, at the end of this chapter, performance measures of the mean monthly differences (MD), the coefficient of correlation (\( r \)) between the observed and the modelled discharge as well as values for the Nash-Sutcliffe criterion (Nash and Sutcliffe 1970) (NS) and the Chiew-McMahon criterion (Chiew and McMahon 1994) (CM) are listed for M1 and all subsequent models.

M1 is defined as the basic initial model concept from which it is started (Table 5.2). Next, variations of climate properties within the catchment region and/or fluctuations within months are included and formulated through different conceptualisation strategies. The aim is to achieve a monthly water balance model with a satisfactory level of performance by employing a step by step model development process.

Figure 5.1: Annual and monthly water balance: Observations and simulated results of model concept M1.
5.3.2 Model Solutions with Temporal Distribution of Monthly Climate Inputs

New model concepts M2 and M3, which are described in Table 5.2 are further developments of M1. They include temporal disaggregation of monthly climate data. Instead of applying solely one single wet ($\langle \bar{t}_{\text{wet}} \rangle$) and dry period ($\langle \bar{t}_{\text{dry}} \rangle$) per month a sequence of alternating precipitation events and dry periods is employed. The long-term mean monthly length of precipitation events ($\langle \bar{t}_{\text{wet}} \rangle$) ranges seasonally between three and four days whereas the length of inter-storm periods ($\langle \bar{t}_{\text{wet}} \rangle$) ranges between four and five days.

Table 5.2: Selection of monthly water balance model concepts M1 - M5.

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Applied model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lumped monthly model input</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>Soil moisture accounting model (Manabe bucket concept), snow module with temperature index algorithm, 2 input variables: mean catchment $\langle P \rangle$ and $\langle T \rangle$.</td>
<td>6: $mf$, $T_i$, $T_0$, $C_y$, $t_e$, $\langle \bar{t}_{\text{wet}} \rangle$.</td>
</tr>
<tr>
<td>Temporal disaggregation of monthly model input</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>Same as M1; furthermore disaggregation of mean catchment $\langle P \rangle$ in alternating storm and inter-storm periods; temporal disaggregation of mean catchment $\langle T \rangle$ through statistical distribution</td>
<td>8: $mf$, $T_i$, $T_0$, $C_y$, $t_e$, $\langle \bar{t}<em>{\text{wet}} \rangle$, $\langle \bar{m} \rangle$, $CV</em>{T,\text{time}}^{1 \rightarrow 12}$.</td>
</tr>
<tr>
<td>M3</td>
<td>Same as M1; furthermore disaggregation of mean catchment $\langle P \rangle$ in alternating storm and inter-storm periods; temporal disaggregation of mean catchment $\langle T \rangle$ through bivariate statistical distribution of precipitation and temperature</td>
<td>10: $mf$, $T_i$, $T_0$, $C_y$, $t_e$, $\langle \bar{t}<em>{\text{wet}} \rangle$, $\langle \bar{m} \rangle$, $CV</em>{T,\text{time}}^{1 \rightarrow 12}$, $CV_{P,\text{time}}^{1 \rightarrow 12}$, $t_{P-T,\text{time}}^{1 \rightarrow 12}$.</td>
</tr>
<tr>
<td>Spatial disaggregation of mean catchment model input</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M4</td>
<td>Same as M1; furthermore spatial disaggregation of mean catchment $\langle T \rangle$ through regression between temperature and elevation</td>
<td>10: $mf$, $T_i$, $T_0$, $C_y$, $t_e$, $\langle \bar{t}<em>{\text{wet}} \rangle$, $H</em>{\text{zone}}$, $f_{H_{\text{zone}}}$, $a$, $b$.</td>
</tr>
<tr>
<td>M5</td>
<td>Same as M1; furthermore spatial disaggregation of mean catchment $\langle P \rangle$ and $\langle T \rangle$ through bivariate statistical distribution</td>
<td>9: $mf$, $T_i$, $T_0$, $C_y$, $t_e$, $\langle \bar{t}<em>{\text{wet}} \rangle$, $CV</em>{T,\text{space}}^{1 \rightarrow 12}$, $CV_{P,\text{space}}^{1 \rightarrow 12}$, $t_{P-T,\text{space}}^{1 \rightarrow 12}$.</td>
</tr>
</tbody>
</table>

In the case of M2, monthly mean catchment temperature $\langle T \rangle$ is statistically distributed within a month (temporal distribution) applying a normal distribution function of catchment mean daily temperature; the relationship presented in Figure A2.4. M2 and M3 include only temporal disaggregation of input data. Both concepts perform very similarly according to the performance measures displayed in Table 5.8 and the monthly discharge plots in Figure 5.2. The improvement of M2 and M3 over M1 is significant. However, the timing of the flow regime is not well reproduced by M2 and M3, even though the peak discharge in May is well matched by the simulations. Both model formulations compute discharge values that are too high for winter and spring (November to April) and too low for the rest of the year. M2 demonstrates that it is not sufficient to solely use distributions which describe the within month distribution of temperature. M3 shows that the same holds true for
temperature and precipitation. Next, the integration of spatially distributed climate variables is tested. Only then, new hypotheses can be made.

5.3.3 Models with Spatial Disaggregation of Lumped Catchment Inputs

Model concepts M4 and M5 are described in Table 5.2. Both models build on mean monthly climate data which are solely spatially distributed. M4 applies a regression function between temperature and elevation and M5 uses a bivariate distribution of temperature and precipitation. The simulated results gained with both models are poor; indeed, the performance measures in Table 5.8 are in the same range as those achieved with M1. Hence, the spatial distribution of climate variables only (without further use of that information in e.g. a snow module which is spatially distributed) is not an emergent property of the monthly water balance.
5.3.4 Models Applying Temporal and Spatial Distributions of Lumped Monthly Inputs

Above, catchment mean monthly climate variables are disaggregated using statistical distributions or regression models which describe either the spatial distribution or the temporal distribution of variables. Now, the following model concepts apply model input data which are disaggregated in space and time. This should then give a clear picture of the dominant scales of temperature and precipitation for simulating the monthly water balance.

M6 combines the temporal disaggregation of precipitation and the distribution of temperature of M2 with the spatial distribution of temperature of M4 using a linear regression model. The performance of M6 is significantly better than derived with any other model concept tested before (Table 5.8, Figure 5.4). The modelled interannual variability of annual yield (Figure 5.4a) and the modelled annual streamflow hydrograph (Figure 5.4b) match the observations very well. The regime curve (Figure 5.4c) and the monthly hydrograph (Figure 5.4d) in particular, nearly follow the observed characteristics, but there is still some potential for further improvements. Simulated discharge in winter is slightly too high and peak flows in May and October are predicted too low if compared with the observed flow regime in Figure 5.4c.

The concept of M7 is similar to M6. It also builds on an alternating sequence of wet and dry periods (alternating precipitation events and inter-storm periods that are parameterised with long-term mean estimates for the length of both periods). But, temperature is spatio-temporally distributed applying a bivariate function
including elevation; the properties of that relationship are presented in Figure A2.8 and Table A2.6. The indices of performance are slightly worse than those of M7 (Table 5.8). The results concerning the regime curve also can be viewed in Figure 5.4e.

M8 relates to the spatio-temporal bivariate relationship between precipitation and temperature (compare with Figure A2.17 and Table A2.14). This model concept combines the features of M3 and M5. The model performance does not benefit from the integration of many statistical relationships packed into one single bivariate function (Figure 5.4f). The accuracy of model results is worse than achieved in models M7, M6 and also M3 (Table 5.8).

---

Annual $Q$, mean monthly $\langle Q \rangle$, and monthly discharge $\langle Q \rangle$.

**Figure 5.4:** Annual and monthly discharge derived with M6 as well as monthly results of M7 and M8.
The model concept M9 is built on an alternating process of storm and inter-storm periods (temporal distribution of precipitation); temperature is spatially distributed by a linear regression model. Spatial distribution of temperature with a linear regression model only (elevation versus temperature) appears to be too simplistic because NS and CM are low and MD is large (Table 5.8).

At this stage, after comprehensive testing of various model concepts (M1-M9) built on either temporal disaggregation, spatial disaggregation of catchment mean monthly climate model input or combinations of both some lessons have been learned. The combination of single statistical relationships (e.g. univariate distribution, regression model) which build on high correlation is advantageous to the application of single but multidimensional statistical distributions. It is important that precipitation is temporally distributed (M2, M6), the spatial distribution of precipitation does not increase model performance (M4, M5, M8). Temperature should either be spatially or temporally distributed (e.g. M2 and M3), a spatial and temporal distribution of temperature does not improve the simulations (e.g. M6 and M7).

Table 5.3: Selection of monthly water balance model concepts M6 - M11.

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Applied model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>M6</td>
<td>Combination of M2 and M4</td>
<td>12: $mf, T_i, T_0, C_{op}, t_e, {\hat{t}<em>i}, {\bar{m}}$, $CV</em>{T,\text{time}}, H_{\text{zone}}, f_{H_{\text{zone}}}, a, b$</td>
</tr>
<tr>
<td>M7</td>
<td>Same as M1; furthermore disaggregation of mean catchment $\langle P \rangle$ in alternating storm and inter-storm periods; spatio-temporal bivariate statistical distribution of catchment mean $\langle T \rangle$ and elevation</td>
<td>10: $mf, T_i, T_0, C_{op}, t_e, {\hat{t}<em>i}, {\bar{m}}$, $CV</em>{T,\text{space-time}}, CV_H, r_{T-H,\text{space-time}}$</td>
</tr>
<tr>
<td>M8</td>
<td>Combination of M3 and M5</td>
<td>10: $mf, T_i, T_0, C_{op}, t_e, {\hat{t}<em>i}, {\bar{m}}$, $CV</em>{T,\text{space-time}}, CV_{P,\text{space-time}}, r_{P-T,\text{space-time}}$</td>
</tr>
<tr>
<td>M9</td>
<td>Same as M1; furthermore disaggregation of catchment mean $\langle P \rangle$ in alternating storm and inter-storm periods; spatial distribution of catchment mean $\langle T \rangle$ through regression between temperature and elevation</td>
<td>11: $mf, T_i, T_0, C_{op}, t_e, {\hat{t}<em>i}, {\bar{m}}$, $H</em>{\text{zone}}, f_{H_{\text{zone}}}, a, b$</td>
</tr>
<tr>
<td>M10</td>
<td>Same as M1; furthermore disaggregation of mean catchment $\langle P \rangle$ in alternating storm and inter-storm periods; spatial disaggregation of mean catchment $\langle T \rangle$ through bivariate statistical distribution of temperature and elevation</td>
<td>10: $mf, T_i, T_0, C_{op}, t_e, {\hat{t}<em>i}, {\bar{m}}$, $CV</em>{T,\text{space}}, CV_H, r_{T-H,\text{space}}$</td>
</tr>
<tr>
<td>M11</td>
<td>Same as M1; furthermore disaggregation of mean catchment $\langle P \rangle$ in alternating storm and inter-storm periods; spatial disaggregation of mean catchment $\langle T \rangle$ through bivariate statistical distribution of precipitation and temperature</td>
<td>10: $mf, T_i, T_0, C_{op}, t_e, {\hat{t}<em>i}, {\bar{m}}$, $CV</em>{T,\text{space}}, CV_{P,\text{space}}, r_{P-T,\text{space}}$</td>
</tr>
</tbody>
</table>

Based on the lessons learnt two new model concepts were developed. M10 temporally disaggregates the mean monthly precipitation with the concept of alternation wet and dry periods and temperature is spatially distributed
through a bivariate distribution of temperature and elevation that are strongly correlated (Table A2.5). The concept of M11 is similar to that of M10 but temperature and precipitation are spatially disaggregated with a bivariate distribution function.

Figure 5.5: Monthly water balance gained with model concept M10.

Figure 5.5a and b shows mean monthly and monthly observed discharge as well as simulated results of M10. The annual results perfectly agree with the observations and are not presented for the sake of brevity. The observed and simulated flow regimes match well even though the primary peak discharge is slightly over-predicted in May and the secondary peak is under-predicted in October (Figure 5.5a). The plot of the monthly hydrograph (Figure 5.5b) demonstrates that low and mean flow is simulated very well but often peak flows as over-predicted which leaves potential for further developments.

The monthly fluctuations of modelled soil moisture and snow water equivalent are displayed in Figure 5.5c. The total soil moisture water holding capacity \( C_{tp} \) is also displayed in the figure as an invariant value over time. The water equivalent stored in the snowpack is plotted on top of \( C_{tp} \) which logically relates to the real world situation (snow on ground). The plot makes it easy to interpret the fluctuations of soil water storage which rises when the snowpack becomes melted. The second peak storage level is generally smaller and arises in October when monthly precipitation is highest (compare 4.7.2).

Up to here, the potential of spatial and temporal distribution of climate inputs for the simulation of the monthly water balance have been tested. Next, the distribution of system state variables (snow water equivalent store and soil moisture storage) is included into the model concept.
5.3.5 Models with Spatial Distribution of Snow Cover Depth and Soil Moisture Storage

All models from M6 to M11, except M9 which showed low values of the Nash-Sutcliffe criterion and the Chiew-McMahon criterion, were adapted, now being called M12 – M16, such that snow depth (snow water equivalent) was spatially distributed according to the spatial distribution of precipitation and temperature. For that adaptation of the model structure no new model parameter is needed (Table 5.4).

Table 5.4: Selection of monthly water balance model concepts M12 - M16.

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Applied model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>M12</td>
<td>Same as M6; furthermore spatial distribution of snow cover depth</td>
<td>12: ( mf, T, T_0, C_{ip}, t_c, { \tilde{m}<em>x }, { \tilde{m} }, CV</em>{T,space-time}^{1-12}, H_{zone}, f_{H,zone}, a, b )</td>
</tr>
<tr>
<td>M13</td>
<td>Same as M7; furthermore spatial distribution of snow cover depth</td>
<td>10: ( mf, T, T_0, C_{ip}, t_c, { \tilde{m}<em>x }, { \tilde{m} }, CV</em>{T,space-time}^{1-12}, r_{T,space-time}^{1-12} )</td>
</tr>
<tr>
<td>M14</td>
<td>Same as M8; furthermore spatial distribution of snow cover depth</td>
<td>10: ( mf, T, T_0, C_{ip}, t_c, { \tilde{m}<em>x }, { \tilde{m} }, CV</em>{T,space-time}^{1-12}, CV_{H,space}^{1-12}, CV_{P,space}^{1-12}, r_{T,space-time}^{1-12} )</td>
</tr>
<tr>
<td>M15</td>
<td>Same as M10; furthermore spatial distribution of snow cover depth</td>
<td>10: ( mf, T, T_0, C_{ip}, t_c, { \tilde{m}<em>x }, { \tilde{m} }, CV</em>{T,space}^{1-12}, CV_{H,space}^{1-12}, CV_{P,space}^{1-12}, r_{T,space}^{1-12} )</td>
</tr>
<tr>
<td>M16</td>
<td>Same as M11; furthermore spatial distribution of snow cover depth</td>
<td>10: ( mf, T, T_0, C_{ip}, t_c, { \tilde{m}<em>x }, { \tilde{m} }, CV</em>{T,space}^{1-12}, CV_{P,space}^{1-12}, r_{T,space}^{1-12} )</td>
</tr>
</tbody>
</table>

The performances of all models improve when accounting for the spatial distribution of snow depth. The models M15 and M16 (out of M10 and M11) perform better than M12, M13 and M14 (further developments of M6, M7 and M8).

Next, the spatial distribution of soil moisture storage is introduced as a result of the distribution of climate input and snowmelt. By including the spatial distribution of soil moisture storage, models M12 - M16 have developed into M17-M21; at this point, all the hydro-meteorological data and system states are spatially distributed and carried-over as such from time step t to t+1. The soil moisture bucket concept is also adapted and field capacity is introduced as an additional threshold level for soil water storage (Table 5.5).
Table 5.5: Selection of monthly water balance model concepts M17-M21.

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Applied model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporal and spatial disaggregation of monthly model input, spatial distribution of snow cover depth and soil moisture storage as well as accounting for two sub-surface flow paths</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M17</td>
<td>Same as M12; furthermore spatial distribution of soil moisture storage and accounting for two sub-surface flow paths</td>
<td>14: $m_f$, $T_1$, $T_0$, $C_y$, $t_c$, ${\ell_{ua}}$, ${\ell_{m}}$, $C V_{T \text{ time}}, H_{zone}$, $h_{zone}$, $a$, $b$, $t_{c,in}$, $t_{c, hf}$</td>
</tr>
<tr>
<td>M18</td>
<td>Same as M13; furthermore spatial distribution of soil moisture storage and accounting for two sub-surface flow paths</td>
<td>12: $m_f$, $T_1$, $T_0$, $C_y$, $t_c$, ${\ell_{ua}}$, ${\ell_{m}}$, $C V_{T \text{ space-time}}, C V_{H}, r_{T-H \text{ space-time}}$, $t_{c,in}$, $t_{c, hf}$</td>
</tr>
<tr>
<td>M19</td>
<td>Same as M14; furthermore spatial distribution of soil moisture storage and accounting for two sub-surface flow paths</td>
<td>12: $m_f$, $T_1$, $T_0$, $C_y$, $t_c$, ${\ell_{ua}}$, ${\ell_{m}}$, $C V_{T \text{ space-time}}, C V_{P \text{ space-time}}, r_{T-P \text{ space-time}}$, $t_{c,in}$, $t_{c, hf}$</td>
</tr>
<tr>
<td>M20</td>
<td>Same as M15; furthermore spatial distribution of soil moisture storage and accounting for two sub-surface flow paths</td>
<td>12: $m_f$, $T_1$, $T_0$, $C_y$, $t_c$, ${\ell_{ua}}$, ${\ell_{m}}$, $C V_{T \text{ space}}, C V_{H}, r_{T-H \text{ space}}$, $t_{c,in}$, $t_{c, hf}$</td>
</tr>
<tr>
<td>M21</td>
<td>Same as M16; furthermore spatial distribution of soil moisture storage and accounting for two sub-surface flow paths</td>
<td>12: $m_f$, $T_1$, $T_0$, $C_y$, $t_c$, ${\ell_{ua}}$, ${\ell_{m}}$, $C V_{T \text{ space}}, C V_{P \text{ space}}, r_{T-P \text{ space}}$, $t_{c,in}$, $t_{c, hf}$</td>
</tr>
</tbody>
</table>

Refer to Chapter 2.7.1 for a description of the adopted model which now accounts for three runoff components, saturation excess runoff ($Q_w$), interflow ($Q_{in}$), and baseflow ($Q_{bf}$). The catchment response times for both subsurface components (interflow: $t_{c,in}$; baseflow: $t_{c,bf}$) are derived from the master recession curve (Figure 5.6) which is compiled from daily discharge measurements taken from the gauge station at Federaun. A description of the estimating procedure for subsurface catchment response times is given in Chapter 2.2.6.
The number of model parameters increases by two; the estimated parameter values of the new parameters are listed in Table 5.6.

Table 5.6: Estimates of model parameters.

<table>
<thead>
<tr>
<th>model parameter estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{fc}$ [mm]</td>
</tr>
<tr>
<td>115</td>
</tr>
<tr>
<td>$t_{c-in}$ [day]</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>$t_{c-hf}$ [day]</td>
</tr>
<tr>
<td>170</td>
</tr>
</tbody>
</table>

For all new model versions the coefficients of correlation increased; M17 and M18 and M19 in particular also show significant improvements of all other performance indices (Table 5.8). The monthly results of M18 are displayed in Figure 5.7. Once the new parameters are introduced, those models using temporal disaggregation of precipitation only (M20 and M21) perform worse than those applying the spatial distribution of precipitation as well (M17, M18 and M19). Before, the spatial distribution of precipitation was considered not to be a dominant property of the monthly water balance. Now, when the model system states are modelled spatially distributed also the spatial distribution of precipitation emerges as an important determinant.

Figure 5.7a shows the modelled flow regime using the Manabe bucket without introducing field capacity as an additional model parameter. In comparison to the observations, the simulated curve is characterised by a
slower increase of discharge in spring (March to May) which results in a discharge that is slightly too high from June to August. The displayed characteristic lead to the hypothesis that the catchment’s response to precipitation and snowmelt is slightly too slow. By introducing field capacity as a threshold above which interflow can occur, the model is capable of computing the long-term mean intra-annual discharge very accurately (Figure 5.7b).

The monthly hydrographs of observations and simulations also correspond well (Figure 5.7c). The monthly soil moisture storage fluctuations and the accumulation as well as depletion of snow cover are illustrated in Figure 5.7d. Again, the model parameters controlling the storage capacities of the soil are assigned constant values over time. This is the case for the total soil moisture capacity and at this point field capacity, too. Soil moisture fluctuates around field capacity; it reaches maximum levels during snowmelt from March to May and in October because of high precipitation. The maximum profile storage capacity is never reached, consequently, saturation excess runoff is never computed. In the case of lumped catchment modelling, this is quite obvious. The interpretation of a totally filled soil moisture bucket would indicate that soil profiles are entirely saturated around the whole catchment which is a rather unrealistic scenario.

The performance of M18 has now reached a satisfactory level with twelve model parameters necessary to parameterise the model and all of them being estimated \textit{a priori}. In the following parameters of M18 are listed (please refer to the list of notations in the beginning with a description of abbreviations): $mf$, $T_r$, $T_b$, $C_{sp}$, $C_{fc}$, $t_{c-in}$, $t_{c-bf}$, $\{t_{in}\}$, $\{\overline{P}\}$, $Cy^{1-12}_{T,space-time}$, $Cy^{1-12}_{F,space-time}$, and $r^{1-12}_{F,space-time}$.
5.3.6 Models Including Spatially Distributed Water Holding Capacity of Soil

A logical expansion of the water balance concept is the introduction of distributed soil moisture storage capacities. Well performing model concepts are adopted which explicitly account for different elevation zones \( (H_{\text{zone}}) \); these are M17 and M18. There is no link to elevation in the formulation of M19, even though temperature and precipitation become disaggregated over space and time. Hence, M19 cannot be adapted by using the spatial distribution of soil water holding capacity in a straightforward way.

For models M17 and M18 nine elevation zones \( (9-1H) \) (Table A2.1) are defined between 500 m a.s.l. and 2500 m a.s.l. The investigation of soil characteristics of the Gail catchment (Table 4.1) indicated that the mean water holding capacity of the total soil profile \( (C_p) \) ranges from around 1000 mm in the valley parts and 0 mm in the peak zone. For the sake of simplicity, the decrease of \( C_p \) was assumed to be linear between those upper and lower limits. In the same manner, field capacity \( (C_f) \) is assumed to range linearly from 400 mm to 0 mm.
When the new features of distributed total bucket capacity and field capacity are included in M17 and M18, now named M22 and M23 (Table 5.7), model performance does not improve. In the case of M23, all performance measures even worsened slightly (Table 5.8). Figure 5.8 displays the annual and monthly results derived by M23. The modelled inter-annual variability of annual discharge and the annual hydrograph correspond (Figure 5.8a and 5.8b) well with the observations in M23 as did all previous model versions. The modelled monthly discharge (Figure 5.8c and 5.8d) is very similar to the results of M18 which does not account for distributed soil characteristics (Figure 5.7). Hence, there is no reason for increasing structural complexity by

Table 5.7: Selection of monthly water balance model concepts M22 and M23.

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Applied model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>M22</td>
<td>Same as M17; furthermore including spatially distributed water holding capacity of soil</td>
<td>$mf$, $T_i$, $T_0$, $C_p$, $t_c$, $\langle \bar{f}<em>a \rangle$, $\langle \bar{m} \rangle$, $CV</em>{T, time}^{1-12}$, $H_{zone}$, $f_{H zone}$, $a$, $b$, $t_{c,in}$, $t_{c,bf}$, $H_{t-g}$</td>
</tr>
<tr>
<td>M23</td>
<td>Same as M18; furthermore including spatially distributed water holding capacity of soil</td>
<td>$mf$, $T_i$, $T_0$, $C_p$, $t_c$, $\langle \bar{f}<em>a \rangle$, $\langle \bar{m} \rangle$, $CV</em>{T, space-time}^{1-12}$, $CV_H$, $r_{T-H, space-time}^{1-12}$, $t_{c,in}$, $t_{c,bf}$, $H_{t-g}$</td>
</tr>
</tbody>
</table>
applying distributed soil parameters. At this stage of model conceptualisation, it is difficult to introduce new processes into the model formulation which still can be parameterised *a priori*. In addition, improvement steps in model performance become very small to the point of being hardly noticeable whereas the balance between structural complexity and model performance worsens. Hence, the step by step process of model conceptualisation progressively including emergent properties of the monthly water balance which have been detected on the way is stopped. Several model are found to reach a satisfactory level of performance. Next, a focus is on the balance between model performance and the number of applied parameters.

Table 5.8: Performance of tested monthly water balance models M1 - M23.

<table>
<thead>
<tr>
<th>Model type</th>
<th>MD</th>
<th>r</th>
<th>NS</th>
<th>CM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lumped monthly model input</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1 (Fig. 5.1)</td>
<td>56.953</td>
<td>0.616</td>
<td>-126.875</td>
<td>-165.533</td>
</tr>
<tr>
<td>Temporal disaggregation of monthly model input</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2 (Fig. 5.2)</td>
<td>32.105</td>
<td>0.702</td>
<td>43.213</td>
<td>37.658</td>
</tr>
<tr>
<td>M3 (Fig. 5.2)</td>
<td>30.934</td>
<td>0.722</td>
<td>45.200</td>
<td>42.940</td>
</tr>
<tr>
<td>Spatial disaggregation of model input</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M4 (Fig. 5.3)</td>
<td>56.828</td>
<td>0.558</td>
<td>-121.117</td>
<td>-127.649</td>
</tr>
<tr>
<td>M5 (Fig. 5.3)</td>
<td>46.086</td>
<td>0.627</td>
<td>-56.553</td>
<td>-53.535</td>
</tr>
<tr>
<td>Temporal and spatial disaggregation of monthly model input:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M6 (Fig. 5.4)</td>
<td>26.081</td>
<td>0.771</td>
<td>58.055</td>
<td>57.082</td>
</tr>
<tr>
<td>M7 (Fig. 5.4)</td>
<td>32.054</td>
<td>0.693</td>
<td>42.956</td>
<td>37.839</td>
</tr>
<tr>
<td>M8 (Fig. 5.4)</td>
<td>32.895</td>
<td>0.686</td>
<td>41.494</td>
<td>35.889</td>
</tr>
<tr>
<td>M9</td>
<td>32.777</td>
<td>0.789</td>
<td>20.500</td>
<td>27.028</td>
</tr>
<tr>
<td>M10 (Fig. 5.5)</td>
<td>27.853</td>
<td>0.795</td>
<td>51.446</td>
<td>50.349</td>
</tr>
<tr>
<td>M11</td>
<td>28.304</td>
<td>0.797</td>
<td>50.347</td>
<td>48.959</td>
</tr>
<tr>
<td>Temporal and spatial disaggregation of monthly model input and spatial distribution of snow cover depth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M12</td>
<td>25.916</td>
<td>0.755</td>
<td>56.836</td>
<td>56.886</td>
</tr>
<tr>
<td>M13</td>
<td>25.930</td>
<td>0.781</td>
<td>57.506</td>
<td>56.069</td>
</tr>
<tr>
<td>M14</td>
<td>25.556</td>
<td>0.791</td>
<td>59.192</td>
<td>58.193</td>
</tr>
<tr>
<td>M15</td>
<td>23.020</td>
<td>0.840</td>
<td>64.648</td>
<td>58.718</td>
</tr>
<tr>
<td>M16</td>
<td>23.831</td>
<td>0.839</td>
<td>64.790</td>
<td>57.676</td>
</tr>
<tr>
<td>Temporal and spatial disaggregation of monthly model input, spatial distribution of snow cover depth and soil moisture storage as well as accounting for two subsurface flow paths</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M17</td>
<td>25.103</td>
<td>0.824</td>
<td>61.511</td>
<td>48.839</td>
</tr>
<tr>
<td>M18 (Fig. 5.7)</td>
<td>23.737</td>
<td>0.902</td>
<td>71.359</td>
<td>56.248</td>
</tr>
<tr>
<td>M19</td>
<td>23.140</td>
<td>0.905</td>
<td>71.823</td>
<td>58.844</td>
</tr>
<tr>
<td>M20</td>
<td>30.110</td>
<td>0.896</td>
<td>52.419</td>
<td>34.931</td>
</tr>
<tr>
<td>M21</td>
<td>28.391</td>
<td>0.905</td>
<td>58.221</td>
<td>40.380</td>
</tr>
<tr>
<td>Temporal and spatial disaggregation of monthly model input, spatial distribution of snow cover depth, soil moisture storage and profile water holding capacity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M22 (Fig. 5.8)</td>
<td>24.679</td>
<td>0.831</td>
<td>63.374</td>
<td>55.788</td>
</tr>
<tr>
<td>M23 (Fig. 5.8)</td>
<td>24.957</td>
<td>0.896</td>
<td>67.491</td>
<td>55.313</td>
</tr>
</tbody>
</table>

*MD...Sum of the mean differences between simulated and observed discharge [1]*  
*NS...Nash-Sutcliffe criterion (Nash and Sutcliffe 1970) [1]*  
*CM...Chiew-McMahon criterion (Chiew and McMahon 1994) [1]*  
*r...Coefficient of correlation [1]*

Figure 5.9 illustrates performance versus number of applied parameters of all monthly water balance models tested. Even though all models appear in the figure, only those performing best and worst are indicated by name.
Both performance measures show the same characteristic; in general, the coefficient of correlation ($r$) increases with more complex models and the mean difference between observed and simulated monthly discharge ($MD$) decreases when new parameters are introduced. Both diagrams indicate that a maximum of model performance is reached using around twelve parameters. Performance decreases slightly for M23, M22, and M17, which are based on thirteen to fifteen parameters. Models M18, M19, and M21 perform best, all of which use twelve parameters.

Figure 5.9: Mean difference $MD$ and coefficient of correlation $r$ between observed and simulated monthly discharge versus number of applied model parameters.

5.4 Summary and Conclusions

The distributions of precipitation and temperature over space and time were analysed in detail. Linear regressions and analytical univariate as well as bivariate distributions were fitted to the empirical data. The regression and distribution functions were parameterised in order to be used for the conceptualisation of monthly water balance models. Those model concepts were tested and results were evaluated by comparing simulated and observed annual and monthly discharge. By adjusting the model concept in a step by step process through hypothesis testing, several versions of well performing monthly water balance models were developed. Only parsimonious model structures were used at all levels of conceptualisation with model parameters being
estimated *a priori* and not adjusted at any time. Finally, model performance was plotted against the number of applied model parameters.

The study showed that models which are based on around twelve parameters perform very well reaching (M18, M19, and M21) values of around 0.9 for the coefficient of correlation between the observed and simulated discharge. The model concepts of M18, M19, and M21 build on the disaggregation of mean catchment precipitation ($\langle P \rangle$) in alternating storm and inter-storm period and soil moisture storage as well as snow water equivalent is modelled spatially distributed. M18 applies a spatio-temporal bivariate distribution of catchment mean temperature ($\langle T \rangle$) and elevation. In M19 mean catchment temperature ($\langle T \rangle$) and precipitation ($\langle P \rangle$) are spatio-temporally disaggregated through a bivariate distribution whereas M21 applies only a spatial disaggregation of mean catchment temperature ($\langle T \rangle$) and precipitation ($\langle P \rangle$) through a bivariate statistical distribution.

It was found that model performance generally increased strongly at the beginning when simple model concepts were adjusted. However, model performance improved with an increasing number of applied parameters only up to a threshold level of around twelve parameters. Hence, there is a trade-off between the number of applied parameters and model performance. Even though introducing new model processes would possibly lead to a better performance, it would prove difficult to apply those models without calibration.
Chapter 6:
Overall Summary and Conclusions

This study has presented several methodologies for developing predictive water balance models, which can be adopted for ungauged catchments in the Alpine regions. The aim was to develop parsimonious annual, monthly and daily water balance models, which use a minimum number of input variables and model parameters. The small set of parameters used serve as surrogate indicators of emergent properties of the hydrology with a high ability to provide the best information at the catchment scale and evolving time scales.

In Chapter 2 the objective of minimum structural model complexity was reached by following a downward approach, starting from the annual time scale and gradually evolving to the monthly and daily time scales, updating the model structure along the way based on the evidences detected in signature plots and in streamflow hydrographs. By defining emergent properties based on the analysis of signature plots and hydrographs the uncertainty about model structure is reduced but never fully resolved. Chapter 2 only concentrated on the reduction of structural uncertainty in the formulation of appropriate models for estimating the water balance at different time scales but the successful performance of the models also rely strongly on the quality of parameter sets and input data. With parsimonious water balance models one can at the very least concentrate more efforts on the estimation of a smaller set of parameter and input data than it would otherwise be the case for a larger set with more complex models. However, Chapter 3 and 4 concentrated on how to deal effectively with uncertainties of parameter and input values - measured at the plot scale and regionalised to the catchment scale.

Chapter 3 has presented the application of a fuzzy, lumped water balance model. The basic principles of fuzzy logic and the associated arithmetic are introduced and it was shown how these can be used to construct the fuzzy water balance model. The model is then used, with realistic estimates of the uncertainty of catchment characteristics and climatic inputs, to estimate the uncertainty of runoff predictions at the annual and monthly time scales. In addition, the model was used to investigate the relative sensitivity of model predictions, in particular predictions of uncertainty of the catchment water balance, to the uncertainty in the various parameters and climatic inputs. This gives the clue as to which of these parameters must be estimated with more confidence in the future, and how increased confidence in model parameters and inputs will increase the confidence in the model predictions. It was found, for example, that the model predictions were much more sensitive to model parameterisations than to climatic inputs. Finally, the model was also used to investigate the effect of decreased model complexity on the accuracy and uncertainty of model predictions. The fuzzy approach can lead to parsimonious models which are capable of predicting not just the mean response, but also confidence in the predictions, in a straightforward manner, without the excessive computational efforts characteristic of traditional stochastic, Monte Carlo procedures.

In Chapter 4 a methodology for estimating the semi-distributed water balance of alpine catchments on monthly and spatial lines was developed. The focus was on the modelling of the spatial distribution of meteorological data and the definition of an appropriate level of catchment disaggregation, allowing effective estimation of distributed model parameters and input data. Distributed water balance computation is constrained
by the spatial scale of parameter and input data. Snowpack accumulation, snowmelt runoff, evapotranspiration
dynamics and fluxes in the upper soil layer were modelled on the sub-regional scale. Surface and subsurface
runoff generation were parameterised on the sub-catchment scale, hence surface runoff, interflow and baseflow
estimates are limited to that scale.

In Chapter 5 the distributions of precipitation and temperature over space and time were analysed in detail.
The aim was to run monthly water balance models with mean monthly input data only, even though monthly
data have been disaggregated in space and time. Linear regressions and analytical univariate as well as bivariate
distributions were fitted to the empirical data. The regression and distribution functions were parameterised in
order to be used for the conceptualisation of monthly water balance models. Those model concepts were tested
and results were evaluated by comparing simulated and observed annual and monthly discharge. By adjusting
the model concept in a step by step process through hypothesis testing, several versions of well performing
monthly water balance models were developed. Finally, the trade-off between the number of applied parameters
and model performance was presented for all model versions.

Even though it is believed that the approaches to model development that is introduced here can be adapted to
any other catchment, the present work only concentrated on the Gail catchment located in the Austrian-Italian
Alps and to the Upper Enns catchment in central Austria with specific regional characteristics. It can only be
seen, therefore, as a first contribution in an attempt to standardise useful hydrological model concepts at different
space and time scales, in different climates, and expressed in terms of emergent properties. This exercise could
be repeated for all practicable spatial scales from several thousand down to a few tens of square kilometres.
Emergent properties should then be defined clearly on a time-space scale matrix, which should serve as the basis
for the formulation of appropriate model structures in the future.
Chapter 7: References


References


7 References


Appendix 1:  
Fuzzy Arithmetic

A1.1 Basic Rules of Fuzzy Arithmetic

Fuzzy arithmetic is easiest to present using the $\alpha$-level set notation (Figure 3.10). For $\tilde{X}(\alpha) = [x^l(\alpha), x^r(\alpha)]$ the lower and upper bounds of $\tilde{X}(\alpha)$ are indicated by $x^l(\alpha)$ and $x^r(\alpha)$, respectively (Özelkan and Duckstein 2001):

Fuzzy Addition

$$\tilde{X}(\alpha) + \tilde{Y}(\alpha) = [x^l(\alpha) + y^l(\alpha), x^r(\alpha) + y^r(\alpha)]$$  \hspace{1cm} (A1.1)

Fuzzy Subtraction

$$\tilde{X}(\alpha) - \tilde{Y}(\alpha) = [x^l(\alpha) - y^r(\alpha), x^r(\alpha) - y^l(\alpha)]$$  \hspace{1cm} (A1.2)

Fuzzy Multiplication

$$\tilde{X}(\alpha) \cdot \tilde{Y}(\alpha) = [x^l(\alpha) \cdot y^l(\alpha), x^r(\alpha) \cdot y^r(\alpha)]$$  \hspace{1cm} (A1.3)

Fuzzy Division

$$\tilde{X}(\alpha) \div \tilde{Y}(\alpha) = [x^l(\alpha) \div y^r(\alpha), x^r(\alpha) \div y^l(\alpha)]$$  \hspace{1cm} (A1.4)

A1.2 Comparison of Two Fuzzy Numbers

In order to ascertain the form of precipitation, i.e., rain or snow, it has to be decided if $\tilde{T} (> \tilde{T}_{\text{crit}}$ or $\tilde{T} (\leq \tilde{T}_{\text{crit}}$. For example, on the 1st of May 1976 $\tilde{T} = (1^\circ C, 2.5^\circ C, 4^\circ C)$ while $\tilde{T}_{\text{crit}} = (0^\circ C, 1^\circ C, 2^\circ C)$, which means that the two fuzzy membership functions overlap. Different approaches for the comparison of fuzzy numbers have been tested such as through defuzzification of both fuzzy numbers into crisp values. The crisp representatives of fuzzy numbers were defined by either the centres of gravity or the values at the highest levels of presumption, with very similar results. The presented results are based on the second concept.
A1.3 Carry-over of System State Uncertainty from Time t to t+1

In this chapter, the carry-over of fuzziness between time steps is achieved by re-scaling the fuzziness of $\hat{S}$ computed at time step $t-1$ into the corresponding value $\hat{S}'$ at time $t$ through the so-called soil moisture ratio, which is defined as:

$$
\hat{r}_s = 1 - \frac{S(t)}{C_{wp}}
$$

(A1.5)

where $S$ and $C_{wp}$ are the defuzzified crisp equivalents of the fuzzy estimates $\hat{S}$ and $\hat{C}_{wp}$. Referring to Figure 3.10, the re-scaling of the fuzziness of soil moisture storage is described, in the usual manner, by:

$$
\hat{S}'(t) = \left[\hat{s}'_{L}, \hat{s}'_{C}, \hat{s}'_{R}\right] = \left[\hat{s}'_{C}, \Delta'(\alpha = 0)'_{L}, \Delta'(\alpha = 0)'_{C}, \Delta'(\alpha = 0)'_{R}\right]
$$

(A1.6)

where $\hat{s}'_{L}$, $\hat{s}'_{C}$, and $\hat{s}'_{R}$ denote the characteristic left (for $\alpha = 0$), centre (for $\alpha = 1$) and right values (for $\alpha = 0$) of a triangular fuzzy number, respectively, and $\hat{s}'_{L}$, $\hat{s}'_{C}$, and $\hat{s}'_{R}$ denote the re-scaled values. The restriction on complete carry-over is achieved by re-scaling the left and right deviations $\Delta'(\alpha)$ of the lower and upper bounds from the centre value) by the soil moisture ratio $\hat{r}_s$. As indicated in Figure 3.10, for a general value of $\alpha$ this re-scaling is represented by:

$$
\Delta'(\alpha) = \Delta'(\alpha) - \Delta'(\alpha) \cdot \hat{r}_s
$$

(A1.7)

$$
\Delta'(\alpha) = \Delta'(\alpha) - \Delta'(\alpha) \cdot \hat{r}_s
$$

(A1.8)

Note that when the bucket is empty, $\hat{r}_s = 1$, whereas when the bucket is full $\hat{r}_s = 0$. Thus, as per Equations A1.7 and A1.8, it can be seen that when the bucket is empty, the model loses all memory of the fuzziness in previous time steps, and there is thus absolutely no carry-over of fuzziness. On the other hand, when the bucket is full, all of the fuzziness from the previous time step is carried over. The approach presented above is next used to scale the fuzziness from $t$ to $t+1$, using the newly computed fuzziness, $\hat{S}'$, and moisture ratio, $\hat{r}_s$, at time $t$; the process continues in this way into the future.
Figure A1.1: Controlled carry-over of uncertainty of soil moisture from time $t$ to $t+1$. 
Appendix 2:
Distribution of Temperature, Precipitation and Elevation

A2.1 Statistical Distribution of Elevation

Figure A2.1: Histogram of distribution of elevation (spatial distribution): Frequency with which elevation falls into an elevation class. 10 classes of equally spaced ranges from the lowest to the highest elevation. Elevations were extracted from a digital elevation model with a cell size of 500x500 m.

Table A2.1: Gail catchment subdivided into nine elevation zones (\(H_{z,9}\)) from 475 m a.s.l. to 2525 m a.s.l.

<table>
<thead>
<tr>
<th>Zone</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean elevation (H_{z,\text{mean}}) [m a.s.l.]</td>
<td>500</td>
<td>750</td>
<td>1000</td>
<td>1250</td>
<td>1500</td>
<td>1750</td>
<td>2000</td>
<td>2250</td>
<td>2500</td>
</tr>
<tr>
<td>Catchment fraction (f_{H_{z,\text{zone}}}) [1]</td>
<td>0.096</td>
<td>0.158</td>
<td>0.161</td>
<td>0.160</td>
<td>0.162</td>
<td>0.141</td>
<td>0.079</td>
<td>0.034</td>
<td>0.009</td>
</tr>
</tbody>
</table>
A2.2 Statistical Analysis of Temperature

A2.2.1 Temperature versus Elevation

Observed mean monthly temperature $\langle T_m \rangle$, exceptionally visualised as $\langle T_m \rangle$ above, and elevation $H$ of monitoring station.

Figure A2.2: Scatter plot of and linear regression ($y = a \cdot x + b$) between observed mean monthly temperature ($y$) versus elevation ($x$). Gail catchment: upstream of Federau. Period: 1971-1995.
A2.2.2 Univariate Statistical Distributions of Temperature

Cumulative frequency $F(\langle T_{st} \rangle')$ of the transformed (Schröder 1969) mean monthly temperature $\langle T_{st} \rangle'$ from 15 temperature monitoring stations (plotting position of empirical data according to Blom (1958)).

**Figure A2.3:** Distribution of temperature within the Gail catchment (spatial distribution) and fitted analytical cumulative normal distribution function. Gail catchment: upstream of Federaun. Period: 1971-1995.

**Table A2.2:** Estimated coefficients of variation ($CV_{T_{space}}^{1-12}$) of the spatial distribution of mean monthly temperature.

<table>
<thead>
<tr>
<th>month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>Mai</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CV_{T_{space}}^{1-12}$</td>
<td>-1.07</td>
<td>-0.78</td>
<td>+1.95</td>
<td>+0.69</td>
<td>+0.34</td>
<td>+0.24</td>
<td>+0.19</td>
<td>+0.17</td>
<td>+0.20</td>
<td>+0.23</td>
<td>+0.76</td>
<td>-2.53</td>
</tr>
</tbody>
</table>
Appendix 2

Cumulative frequency $F(T_{cmt})$ of catchment mean daily temperature $T_{cmt}$ (plotting position of empirical data according to Blom (1958)).

Figure A2.4: Distribution of temperature within a month (temporal distribution) and fitted analytical cumulative normal distribution function. Gail catchment: upstream of Federaun. Period: 1971-1995.

Table A2.3: Estimated coefficients of variation ($CV_{T_{cmt}}^{1-12}$) of the temporal distribution of catchment mean daily temperature.

<table>
<thead>
<tr>
<th>month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CV_{T_{cmt}}^{1-12}$</td>
<td>-1.49</td>
<td>-2.22</td>
<td>+5.05</td>
<td>+0.86</td>
<td>+0.41</td>
<td>+0.28</td>
<td>+0.19</td>
<td>+0.19</td>
<td>+0.28</td>
<td>+0.50</td>
<td>+2.69</td>
<td>-2.44</td>
</tr>
</tbody>
</table>
Cumulative frequency $F(T_{st})$ of observed mean daily temperature from 15 temperature monitoring stations $T_{st}$ (plotting position of empirical data according to Blom (1958)).


Table A2.4: Estimated coefficients of variation ($CV_{T,space-time}^{1-12}$) of the spatio-temporal distribution of mean daily temperature.

<table>
<thead>
<tr>
<th>month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>Mai</th>
<th>Jun</th>
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<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CV_{T,space-time}^{1-12}$</td>
<td>-1.74</td>
<td>-3.67</td>
<td>+2.62</td>
<td>+0.86</td>
<td>+0.46</td>
<td>+0.33</td>
<td>+0.25</td>
<td>+0.25</td>
<td>+0.34</td>
<td>+0.56</td>
<td>+2.36</td>
<td>-2.83</td>
</tr>
</tbody>
</table>
A2.2.3 Bivariate Statistical Distributions of Temperature and Elevation

Plotting position of empirical data according to Blom (1958).

Figure A2.6: Cumulative frequency \( F(H') \) of the transformed (Schröder 1969) elevation \( H' \) and fitted analytical cumulative normal distribution function. Gail catchment: upstream of Federaun.

Table A2.5: Estimated coefficients of correlation \( r_{T-\text{space}}^{1-12} \) of spatially distributed mean monthly temperature and elevation.

<table>
<thead>
<tr>
<th>month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>Mai</th>
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<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+0.39</td>
<td>-0.53</td>
<td>-0.93</td>
<td>-0.98</td>
<td>-0.99</td>
<td>-0.99</td>
<td>-0.98</td>
<td>-0.97</td>
<td>-0.86</td>
<td>-0.26</td>
<td>+0.44</td>
<td></td>
</tr>
</tbody>
</table>

Table A2.6: Estimated coefficients of correlation \( r_{T-\text{space-time}}^{1-12} \) of spatially distributed mean daily temperature and elevation.

<table>
<thead>
<tr>
<th>month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>Mai</th>
<th>Jun</th>
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<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.31</td>
<td>-0.48</td>
<td>-0.41</td>
<td>-0.40</td>
<td>-0.42</td>
<td>-0.44</td>
<td>-0.43</td>
<td>-0.40</td>
<td>-0.35</td>
<td>-0.44</td>
<td>-0.35</td>
<td></td>
</tr>
</tbody>
</table>
Figure A2.7: Distribution of temperature and elevation within the catchment (spatial distribution): (a) empirical relationship $\langle T_u \rangle$; $H'$ and; (b) standardised bivariate probability normal density function $f(T_u, H', r)$. Gail catchment: upstream of Federaun. Period: 1971-1995.
Figure A2.8: Distribution of temperature and elevation within a month and the catchment (spatio-temporal distribution): (a) empirical relationship ($T_{st}, H'$); (b) standardised bivariate probability normal density function ($f(T_{st}, H', r)$). Gail catchment: upstream of Federaun. Period: 1971-1995.
A2.3 Statistical Analysis of Precipitation

A2.3.1 Precipitation versus Elevation

Observed mean monthly precipitation $\langle P_{st} \rangle$, exceptionally visualised as $\langle P_{st} \rangle$ above, and elevation $H$ of monitoring station.

Figure A2.9: Scatter plot of and linear regression ($y = a \cdot x + b$) between observed mean monthly precipitation ($y$) versus elevation ($x$). Gail catchment: upstream of Federaun. Period: 1971-1995.
A2.3.2 Univariate Statistical Distributions of Precipitation

Cumulative frequency $F(\langle P_{st} \rangle)$ of the log-transformed (and according to Schröder 1969) observed mean monthly precipitation $\langle P_{st} \rangle$ from 39 precipitation monitoring stations (plotting position of empirical data according to Blom (1958)).

Figure A2.10: Distribution of precipitation within the catchment (spatial distribution) and fitted analytical cumulative normal distribution function. Gail catchment: upstream of Federaun. Period: 1971-1995.

Table A2.7: Estimated coefficients of variation ($CV_{P_{space}}^{1-12}$) of the spatial distribution of mean monthly precipitation.

<table>
<thead>
<tr>
<th>month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>Mai</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CV_{P_{space}}^{1-12}$</td>
<td>0.061</td>
<td>0.055</td>
<td>0.049</td>
<td>0.048</td>
<td>0.044</td>
<td>0.032</td>
<td>0.032</td>
<td>0.027</td>
<td>0.045</td>
<td>0.048</td>
<td>0.048</td>
<td>0.054</td>
</tr>
</tbody>
</table>
Appendix 2

Figure A2.11: Distribution of precipitation within a month (temporal distribution) and fitted analytical cumulative Gamma-distribution function. Gail catchment: upstream of Federaun. Period: 1971-1995.

Table A2.8: Estimated coefficients of variation (\(CV_{\text{time}}^{1-12}\)) of the temporal distribution of catchment daily precipitation.

<table>
<thead>
<tr>
<th>month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>Mai</th>
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<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>(CV_{\text{time}}^{1-12})</td>
<td>0.620</td>
<td>0.633</td>
<td>0.601</td>
<td>0.551</td>
<td>0.587</td>
<td>0.560</td>
<td>0.523</td>
<td>0.571</td>
<td>0.620</td>
<td>0.511</td>
<td>0.529</td>
<td>0.552</td>
</tr>
</tbody>
</table>
Appendix 2

Cumulative frequency $F(P_{st})$ of observed daily precipitation from 39 precipitation monitoring stations $P_{st}$ (plotting position of empirical data according to Blom (1958)).


Table A2.9: Estimated coefficients of variation ($CV_{P_{space-time}}^{1-12}$) of the spatio-temporal distribution of daily precipitation.

<table>
<thead>
<tr>
<th>month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
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<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CV_{P_{space-time}}^{1-12}$</td>
<td>0.574</td>
<td>0.579</td>
<td>0.535</td>
<td>0.540</td>
<td>0.546</td>
<td>0.524</td>
<td>0.492</td>
<td>0.511</td>
<td>0.534</td>
<td>0.510</td>
<td>0.514</td>
<td>0.531</td>
</tr>
</tbody>
</table>
A2.3.3 Bivariate Statistical Distributions of Precipitation and Elevation

Table A2.10: Estimated coefficients of correlation ($r_{p-H_{\text{space}}}$) of spatially distributed mean monthly precipitation and elevation.

<table>
<thead>
<tr>
<th>month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>Mai</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{p-H_{\text{space}}}$</td>
<td>+0.20</td>
<td>+0.23</td>
<td>+0.20</td>
<td>+0.17</td>
<td>+0.24</td>
<td>+0.23</td>
<td>+0.37</td>
<td>+0.34</td>
<td>+0.05</td>
<td>+0.09</td>
<td>+0.01</td>
<td>+0.06</td>
</tr>
</tbody>
</table>

Table A2.11: Estimated coefficients of correlation ($r_{p-H_{\text{space-time}}}$) of spatially distributed daily precipitation and elevation.

<table>
<thead>
<tr>
<th>month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>Mai</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{p-H_{\text{space-time}}}$</td>
<td>-0.03</td>
<td>-0.01</td>
<td>-0.04</td>
<td>-0.02</td>
<td>-0.01</td>
<td>+0.01</td>
<td>+0.00</td>
<td>+0.02</td>
<td>-0.01</td>
<td>-0.02</td>
<td>-0.04</td>
<td>-0.03</td>
</tr>
</tbody>
</table>
(a) Iso-lines $\alpha$ (0.99, 0.9, 0.5 and 0.1) of fitted bivariate probability normal density function $f(P, H', r)$.

(b) Log-transformed mean monthly precipitation from 39 precipitation monitoring stations $\{P_n\}$, exceptionally visualised as $\{P_n\}$ above, versus transformed (Schröder 1969) elevation of monitoring stations $H'$.

Figure A2.13: Distribution of precipitation and elevation within the catchment (spatial distribution): (a) empirical relationship $\langle P, X, H' \rangle$; (b) standardised bivariate probability normal density function $f(P, H', r)$. Gail catchment: upstream of Federaun. Period: 1971-1995.
Appendix 2

Figure A2.14: Distribution of precipitation and elevation within a month and the catchment (spatio-temporal distribution): (a) empirical relationship \((P_{st}', H, r)\); (b) standardised bivariate probability normal density function \((f(P_{st}', H', r))\). Gail catchment: upstream of Federaun. Period: 1971-1995.

(a) Iso-lines \(\alpha\) (0.99, 0.9, 0.5 and 0.1) of fitted bivariate probability normal density function \(f(P_{st}', H', r)\).

(b) Transformed (Gamma- to normal distribution) observed daily precipitation from 39 precipitation monitoring stations \(P_{st}'\), versus transformed (Schröder 1969) elevation of monitoring stations \(H'\).
A2.4 Bivariate Statistical Distributions of Combined Precipitation and Temperature

Table A2.12: Estimated coefficients of correlation ($r_{P-T,space}^{1-12}$) of spatially distributed mean monthly precipitation and temperature.

<table>
<thead>
<tr>
<th>month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>Mai</th>
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<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{P-T,space}^{1-12}$</td>
<td>-0.19</td>
<td>-0.66</td>
<td>-0.65</td>
<td>-0.56</td>
<td>-0.60</td>
<td>-0.58</td>
<td>-0.72</td>
<td>-0.77</td>
<td>-0.41</td>
<td>-0.60</td>
<td>-0.56</td>
<td>-0.23</td>
</tr>
</tbody>
</table>

Table A2.13: Estimated coefficients of correlation ($r_{P-T,time}^{1-12}$) of catchment daily precipitation and temperature.

<table>
<thead>
<tr>
<th>month</th>
<th>Jan</th>
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<th>Mar</th>
<th>Apr</th>
<th>Mai</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{P-T,time}^{1-12}$</td>
<td>+0.18</td>
<td>+0.04</td>
<td>+0.01</td>
<td>-0.14</td>
<td>-0.16</td>
<td>-0.20</td>
<td>-0.01</td>
<td>+0.12</td>
<td>-0.04</td>
<td>-0.04</td>
<td>+0.10</td>
<td>+0.22</td>
</tr>
</tbody>
</table>

Table A2.14: Estimated coefficients of correlation ($r_{P-T,space-time}^{1-12}$) of spatially distributed daily precipitation and temperature.

<table>
<thead>
<tr>
<th>month</th>
<th>Jan</th>
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<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{P-T,space-time}^{1-12}$</td>
<td>+0.12</td>
<td>+0.07</td>
<td>-0.07</td>
<td>-0.15</td>
<td>-0.16</td>
<td>-0.03</td>
<td>-0.04</td>
<td>+0.08</td>
<td>-0.06</td>
<td>-0.06</td>
<td>+0.13</td>
<td>+0.15</td>
</tr>
</tbody>
</table>
Figure A2.15: Distribution of precipitation and temperature within the catchment (spatial distribution): (a) empirical relationship \( \langle P_a \rangle, \langle T_a \rangle \); (b) standardised bivariate probability normal density function \( f(\langle P_a \rangle, \langle T_a \rangle; r) \). Gail catchment: upstream of Federaun. Period: 1971-1995.
Figure A2.16: Distribution of precipitation and temperature within a month (temporal distribution): (a) empirical relationship ($P_{\text{cnt}}^*, T_{\text{cnt}}$), (b) standardised bivariate probability normal density function ($f(P_{\text{cnt}}^*, T_{\text{cnt}}^*)$). Gall catchment: upstream of Federaun. Period: 1971-1995.
Figure A2.17: Distribution of precipitation and temperature within a month and the catchment (spatio-temporal distribution): (a) empirical relationship ($P_{st}^\prime$, $T_{st}$); (b) standardised bivariate probability normal density function ($f(P_{st}^\prime, T_{st}, r)$). Gail catchment: upstream of Federaun. Period: 1971-1995.