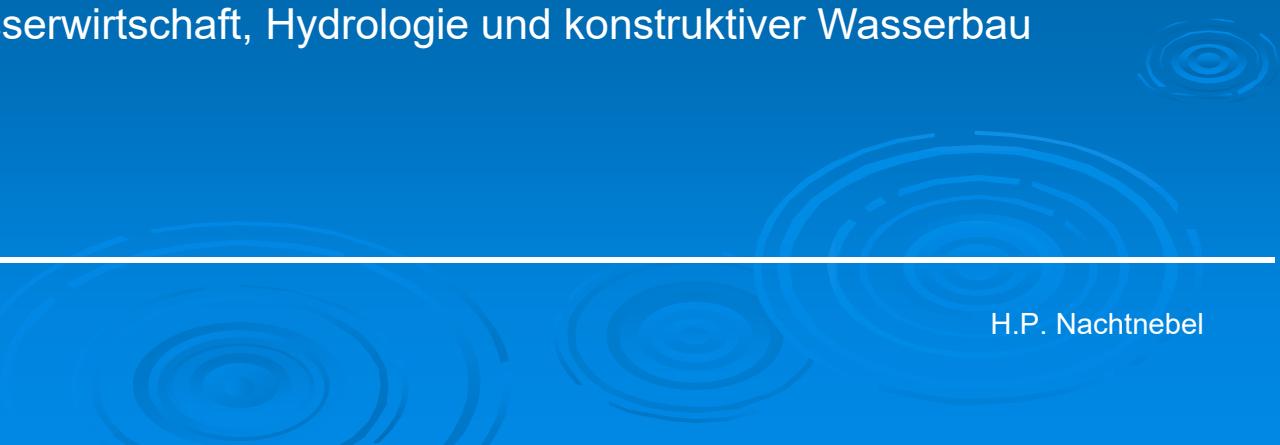

Unit 3: Identification of Hazardous Events

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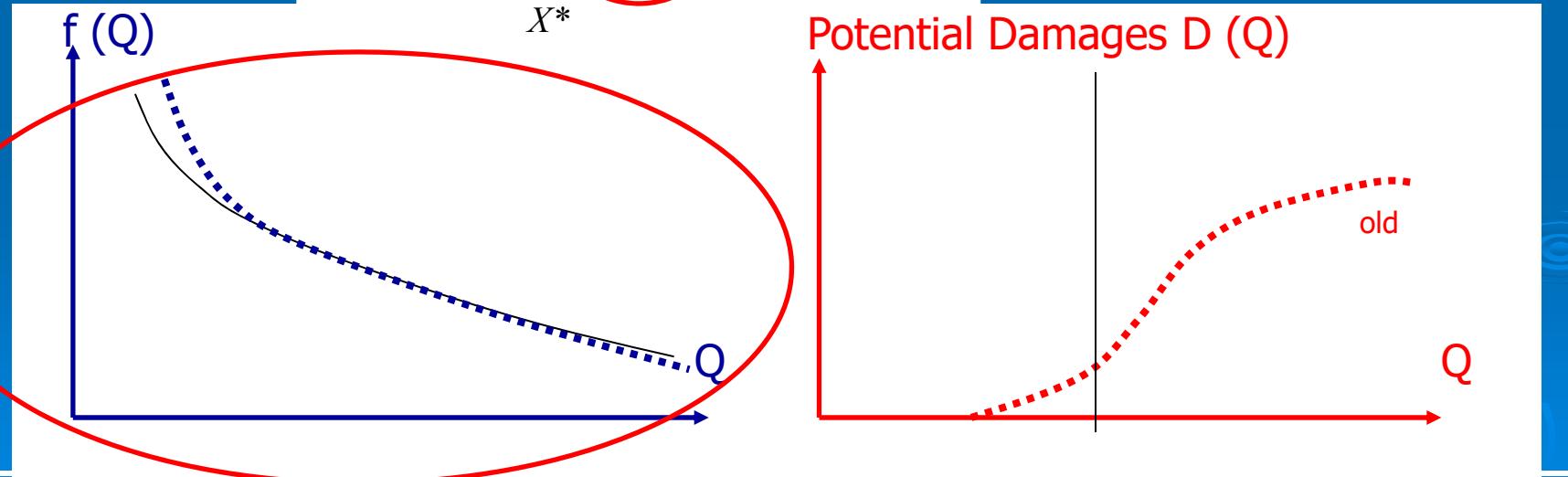
Objectives

- Risk assessment is based on the
 - estimation of probabilities of hazardous events
 - estimation of a loss (damage) function for env. risk
 - or dose-response function for health risk
- This module analysis probabilities of hazardous events

Risk Definition

- A hazardous event X
- A probability distribution function (pdf) with $f(X)$
- The consequences (damages, victims,...) D(X)

$$R(X^*) = \int_{X^*}^{\infty} f(X) \cdot D(X) \cdot dX$$



Identification of Extremes

- The main task is to analyse a time series
 - Continuous observations $x(t)$
 - Discrete observations $x_i(t_i + \Delta t)$
- Detection of hazardous events
- Establishing a statistics

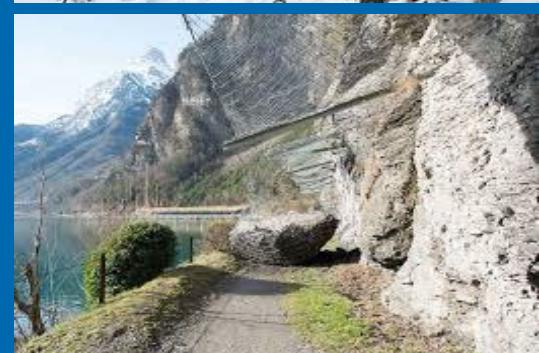
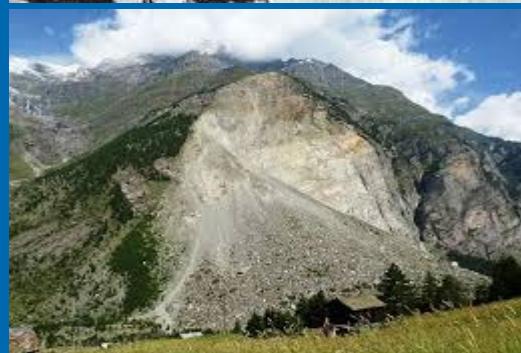
Identification of Extremes

- Extreme events

- Avalanches



- Rockfall



- Mudflow



Identification of Extremes

- Hail storms



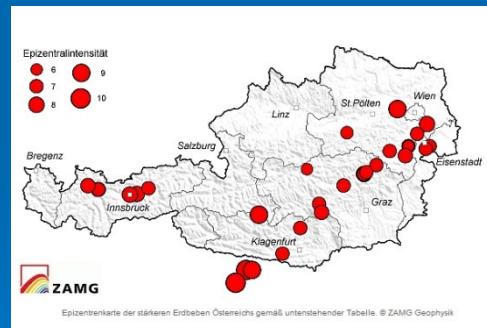
- Lightnings



- Forest fires



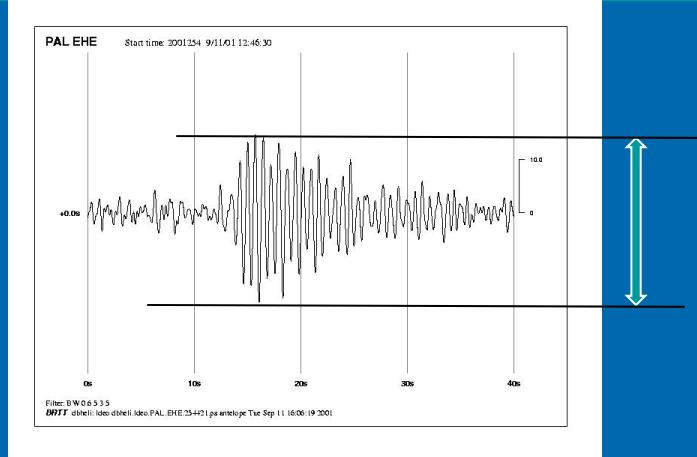
- Earthquakes



- Floods

Identification of Extremes

➤ Earthquakes



➤ Heavy rainfall events

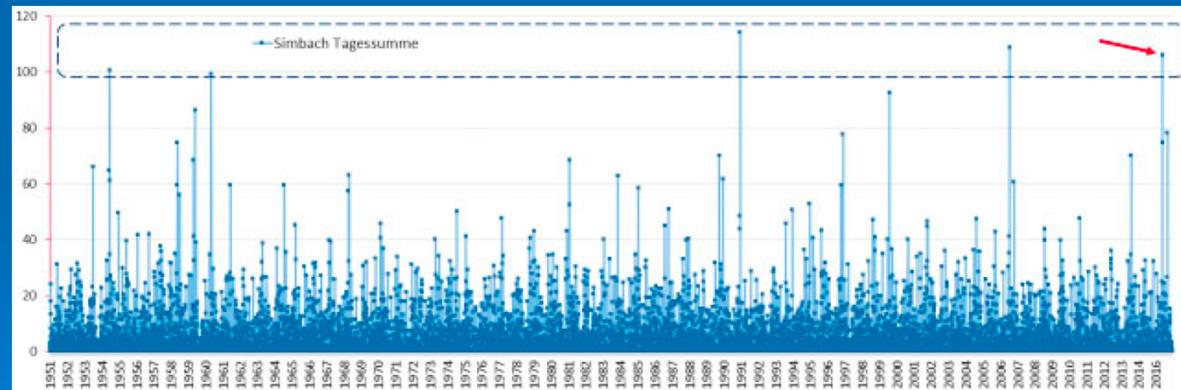
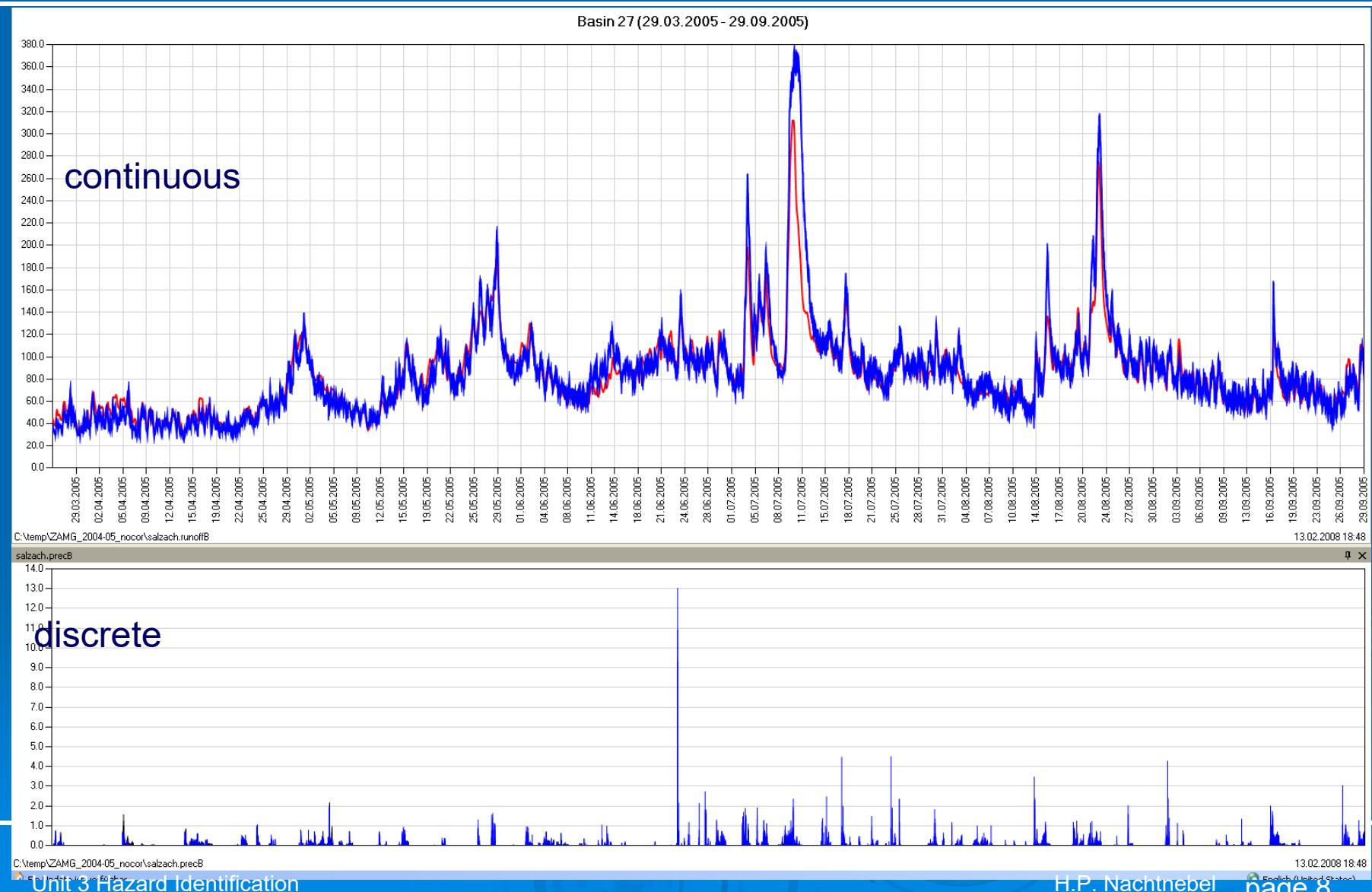


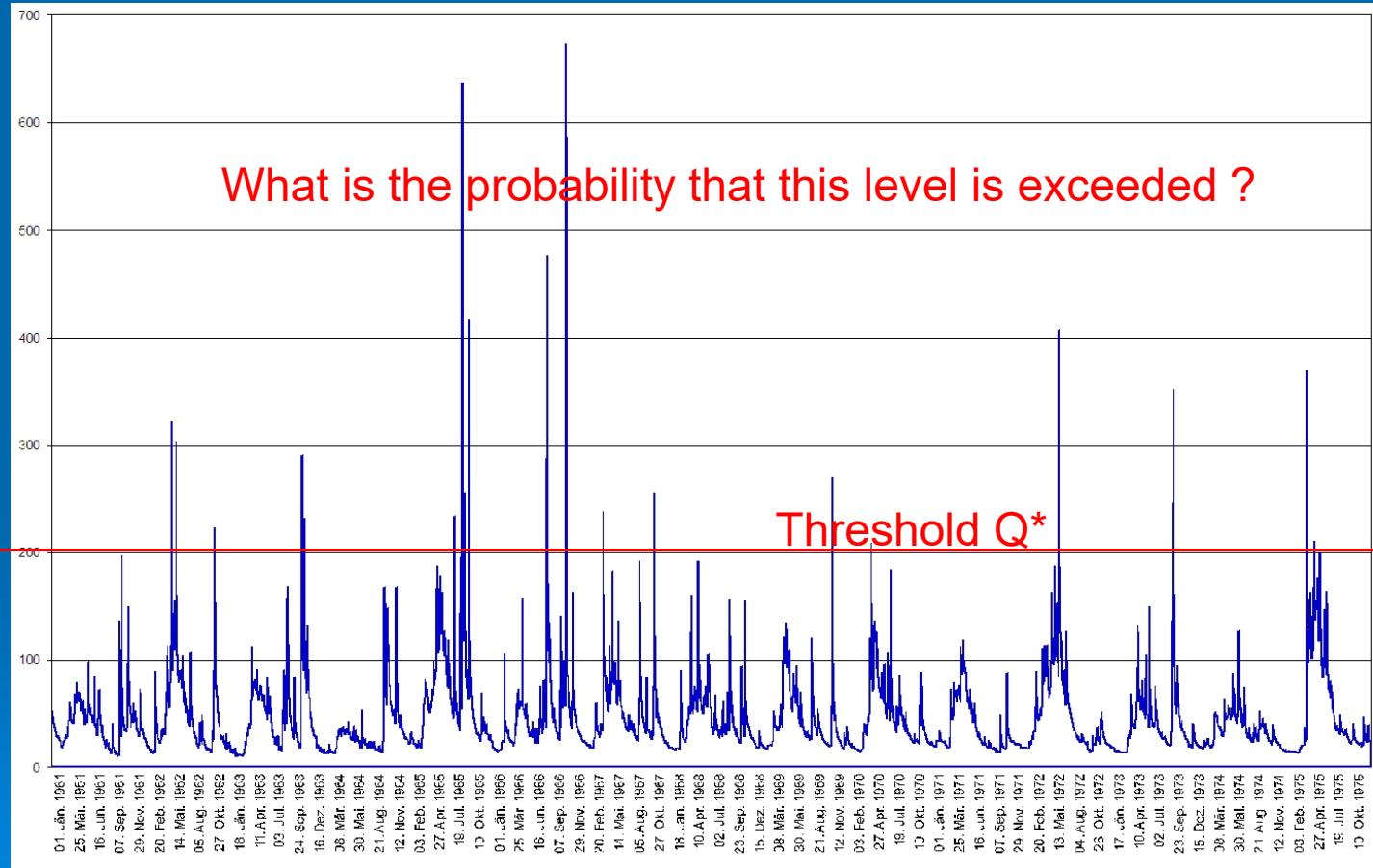
Bild 23 [9] Niederschläge Simbach Tageswerte 1951 – 2016. Vom Autor aus den DWD-Daten und einer Ergänzung für den 1.6.2016 erstellt

Time Series of Runoff and Precipitation

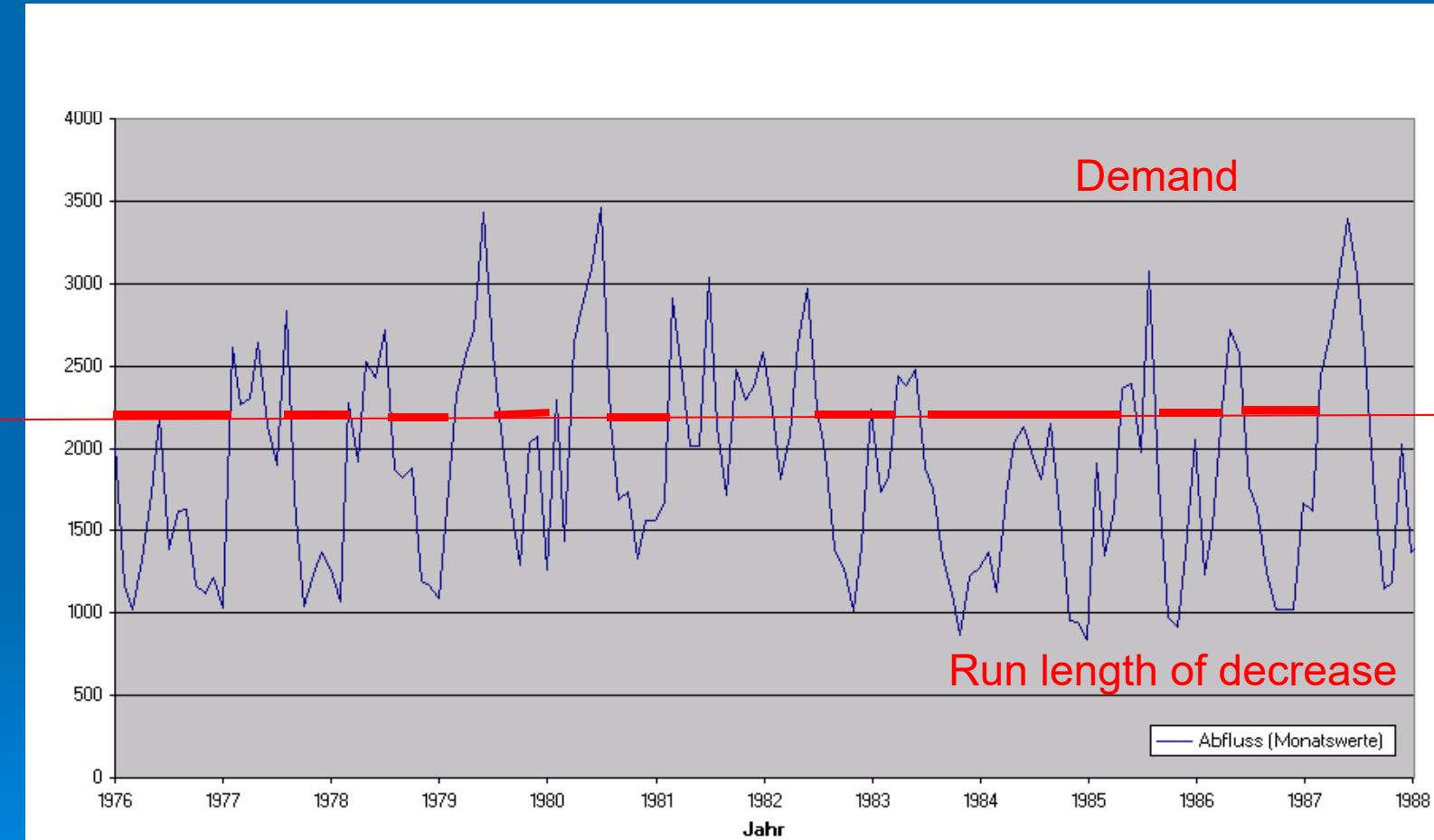


An Example

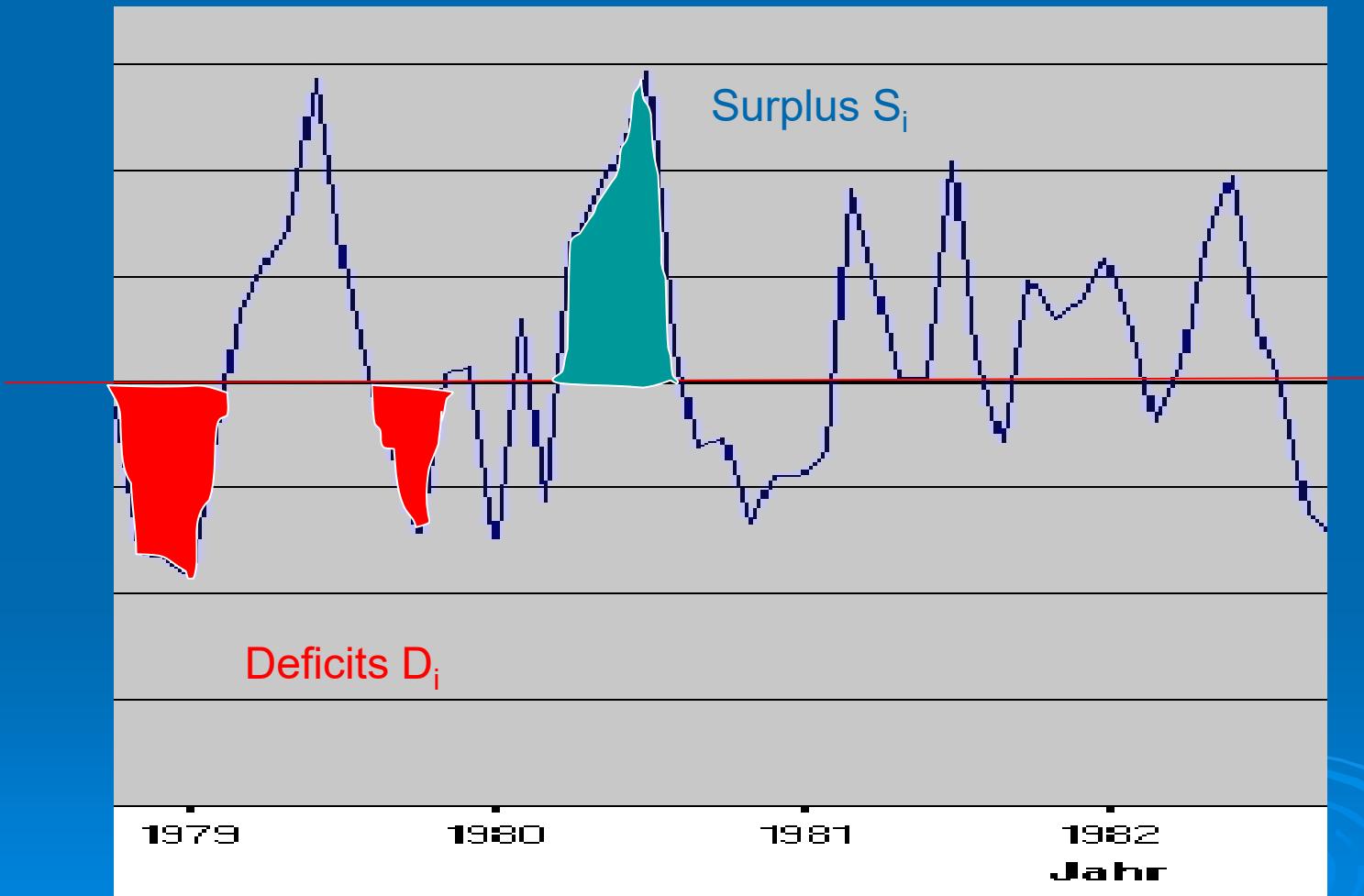
- A critical load has to be analysed



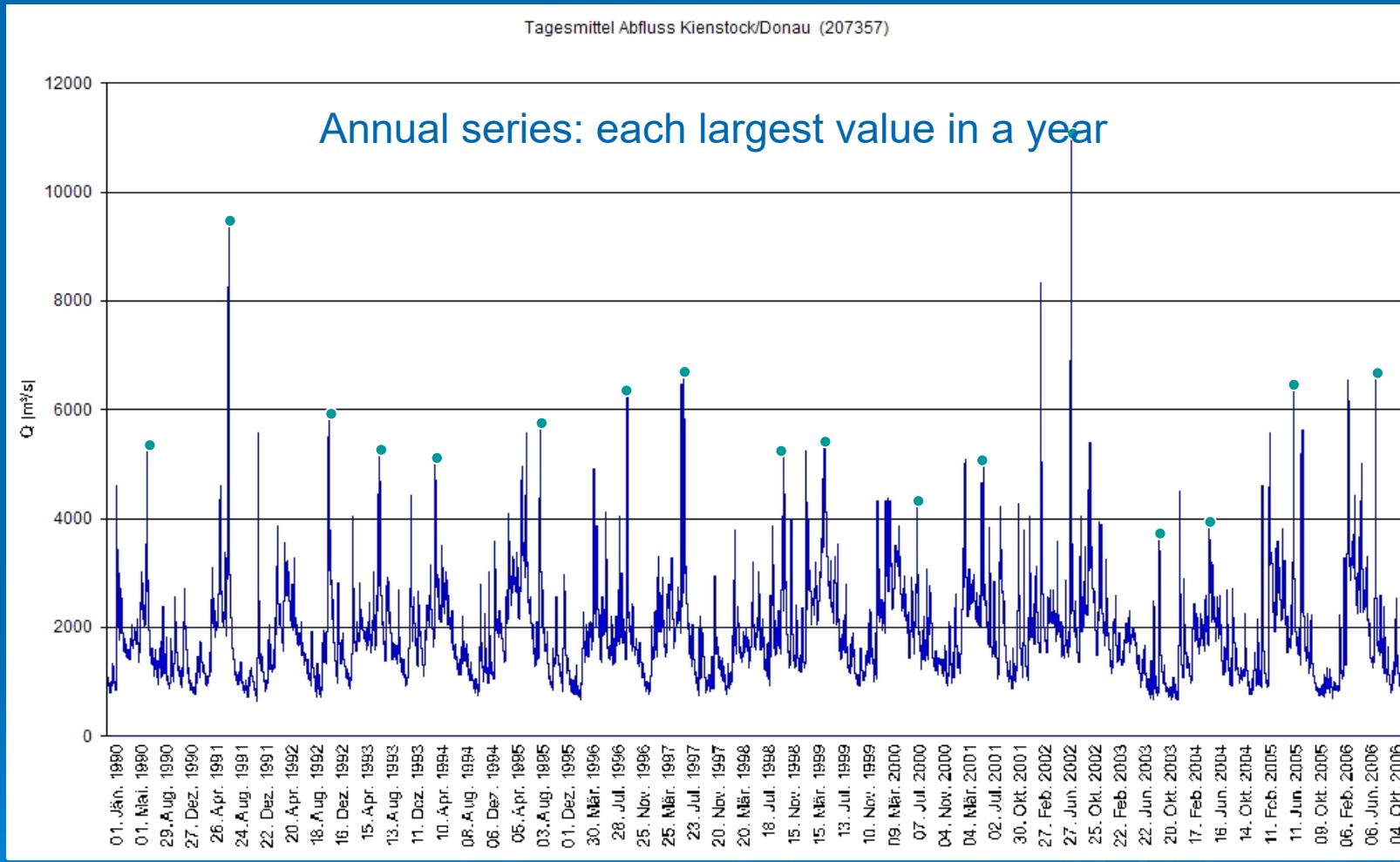
An Example: temporal variability of water availability



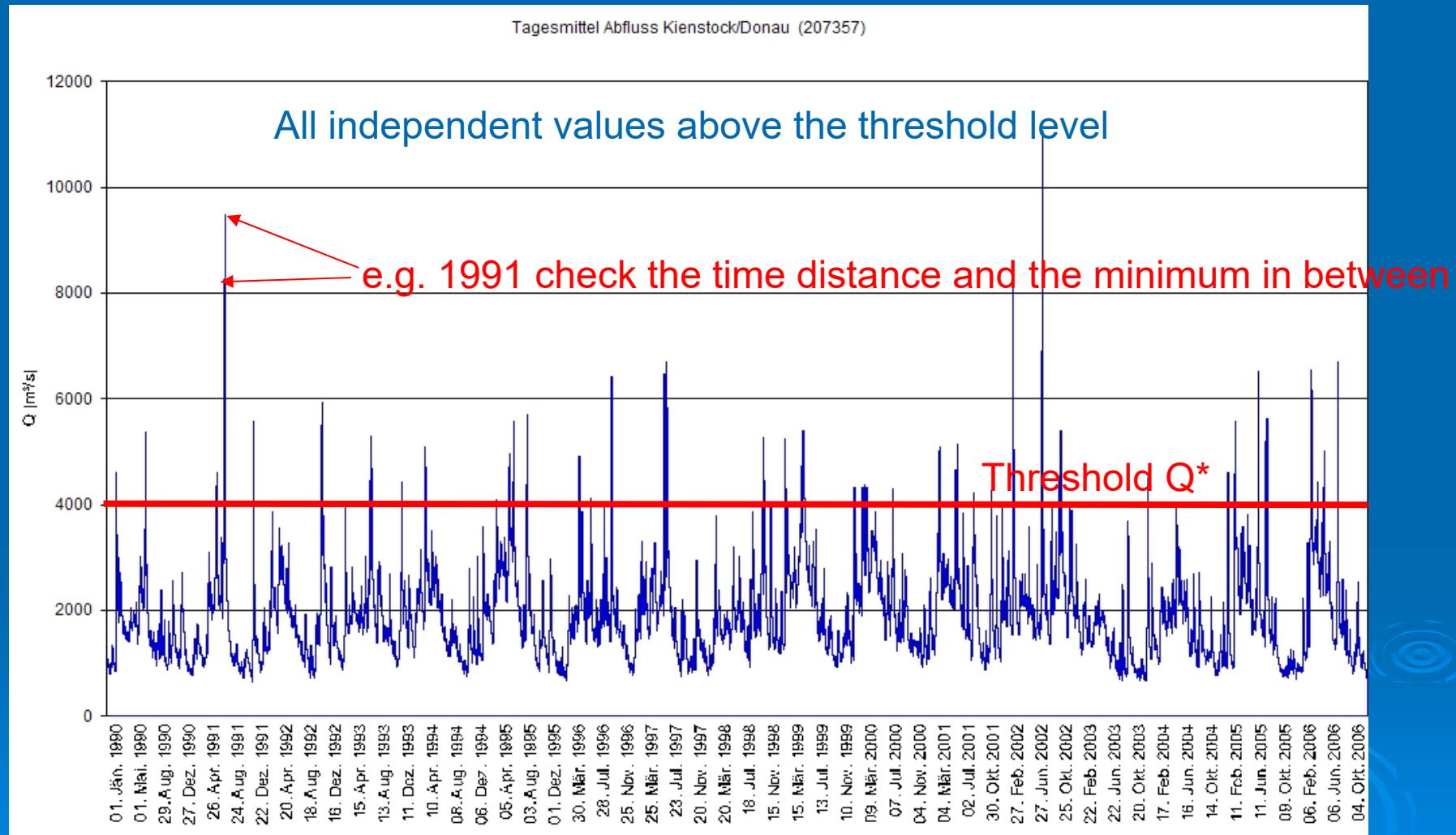
An Example: surplus and deficit



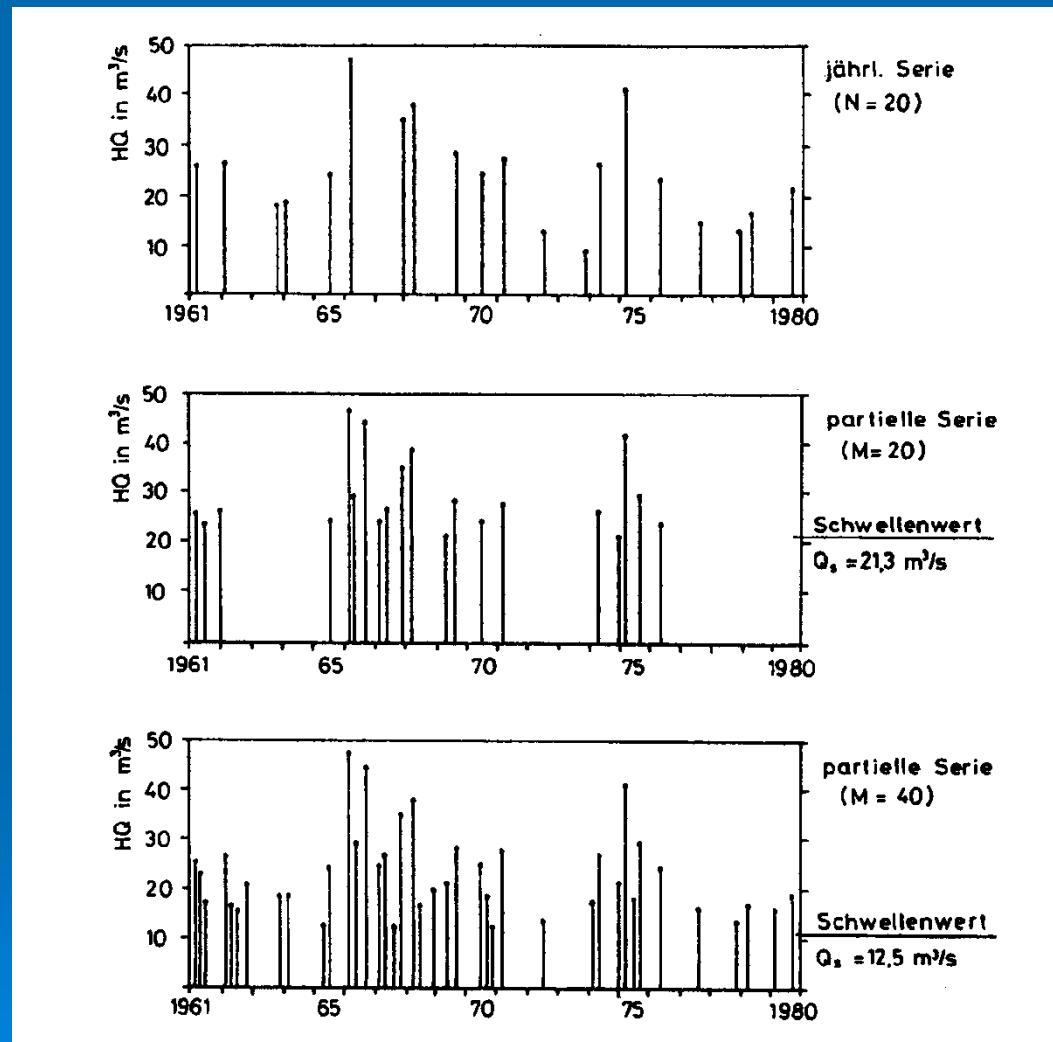
Extremes



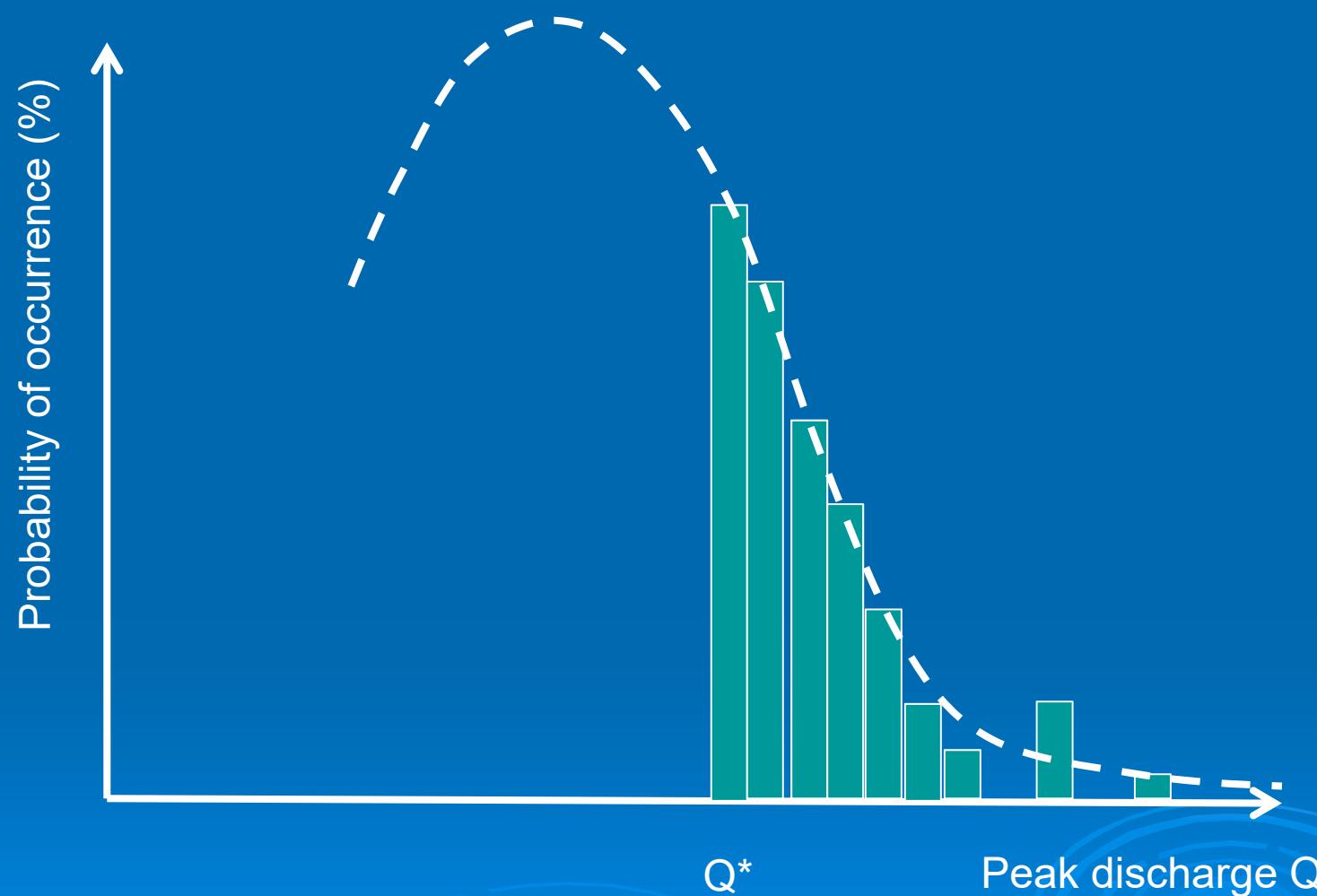
Partial Duration Series



A comparison of annual and partial series



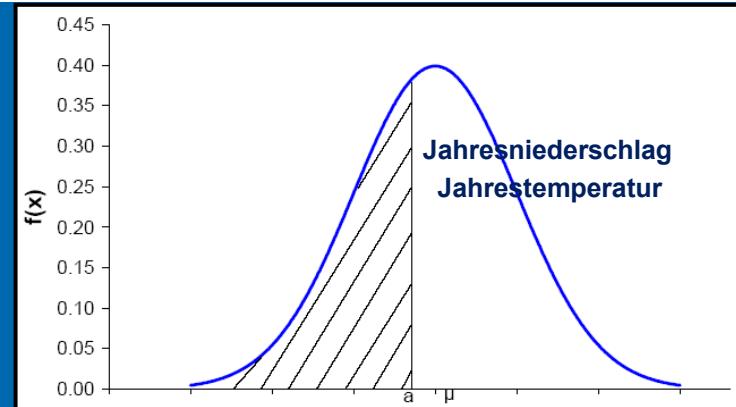
Distribution of selected flood peaks



Example of 2 distributions

➤ Normal distribution

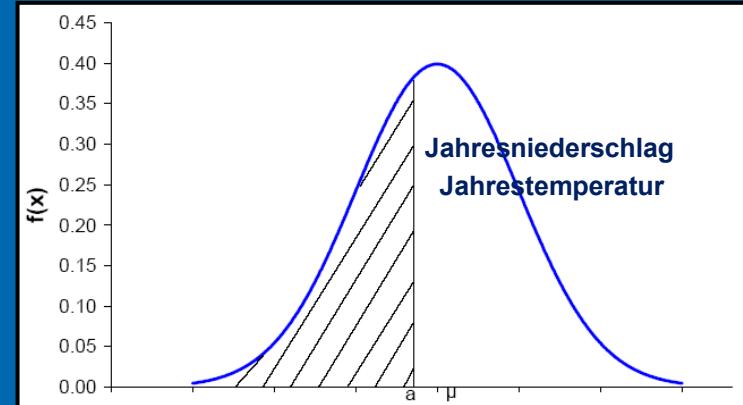
- 2-parameters
- Symetric
- Unbounded on both sides



Example of 2 distributions

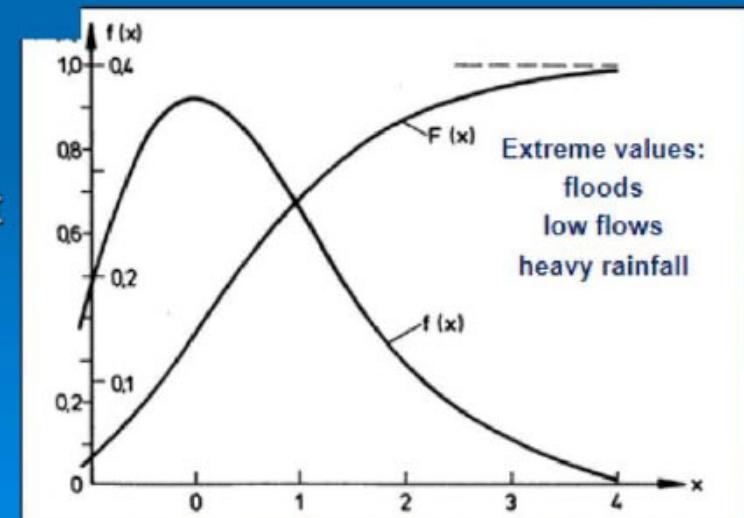
➤ Normal distribution

- 2-parameters
- Symetric
- Unbounded on both sides



➤ Gumbel distribution

- 2-parametric
- Double exponentiel $F(x) = e^{-e^{-\frac{a+x}{c}}}$
- Asymmetric with fixed skewness
- $c_s = 1,1396$
- a – Location parameter, Modalwert
$$a = \bar{x} - 0,5772 * c$$
- c – scale parameter
$$c = \frac{1,28255}{s_x}$$
- Unbounded at the right side



Useful Distributions for Extremes

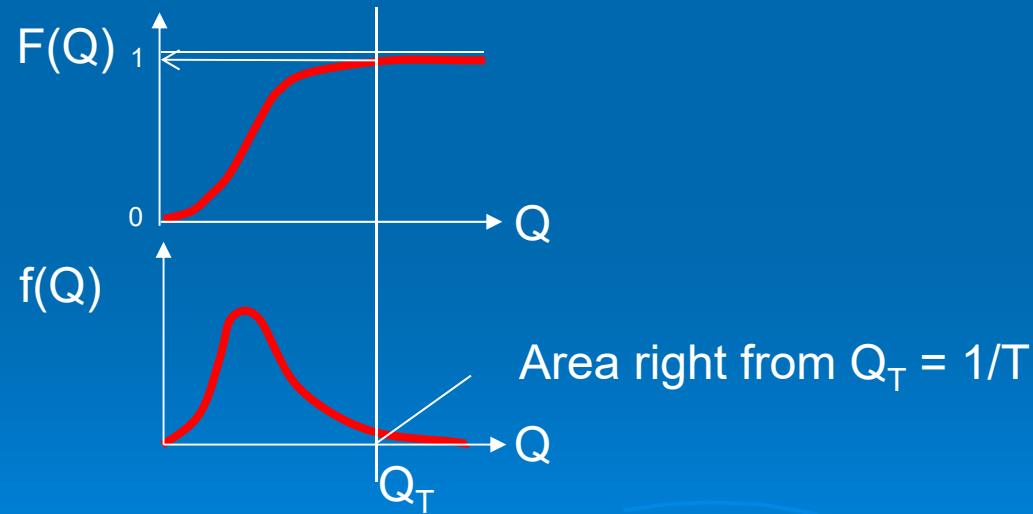
- Log-Normaldistribution
- Gumbeldistribution
- Log-Gumbeldistribution
- Pearson III-distribution
- Log-Pearson III distribution
- Weibull distribution
- Wakeby distribution
- Gamma distribution
-

Quantils and distribution

➤ Relation between Q and $P(Q > Q_T)$

$$P(Q \leq Q_T) = 1 - 1/T = F(Q) \quad P(Q > Q_T) = 1 - P(Q \leq Q_T)$$

- $F(Q)$ is the distribution function
- $f(Q)$ is the density function



Gumbel distribution

- 2 parameters: a, c
- Double exponential
- Left side bounded, right side unbounded

$$F(x_T) = e^{-e^{\frac{-a+xT}{c}}} = 1 - \frac{1}{T} \Rightarrow \text{Take the log}$$

Gumbel distribution

- 2 parameters: a, c
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$$-e^{\frac{-a+x_T}{c}} = \ln\left(1 - \frac{1}{T}\right) \Rightarrow \text{Take the log and multiply by } (-1)$$

Gumbel distribution

- 2 parameters: a, c
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$$F(x_T) = e^{-e^{\frac{-a+x_T}{c}}} = 1 - \frac{1}{T} \Rightarrow \text{Take the log}$$

$$-e^{\frac{-a+x_T}{c}} = \ln\left(1 - \frac{1}{T}\right) \Rightarrow \text{Take the log and multiply by } (-1)$$

$$\frac{-a+x_T}{c} = \ln\left\{-\ln\left(1 - \frac{1}{T}\right)\right\} = y_T \quad \text{is a straight line}$$

Example: Estimation of a rare event

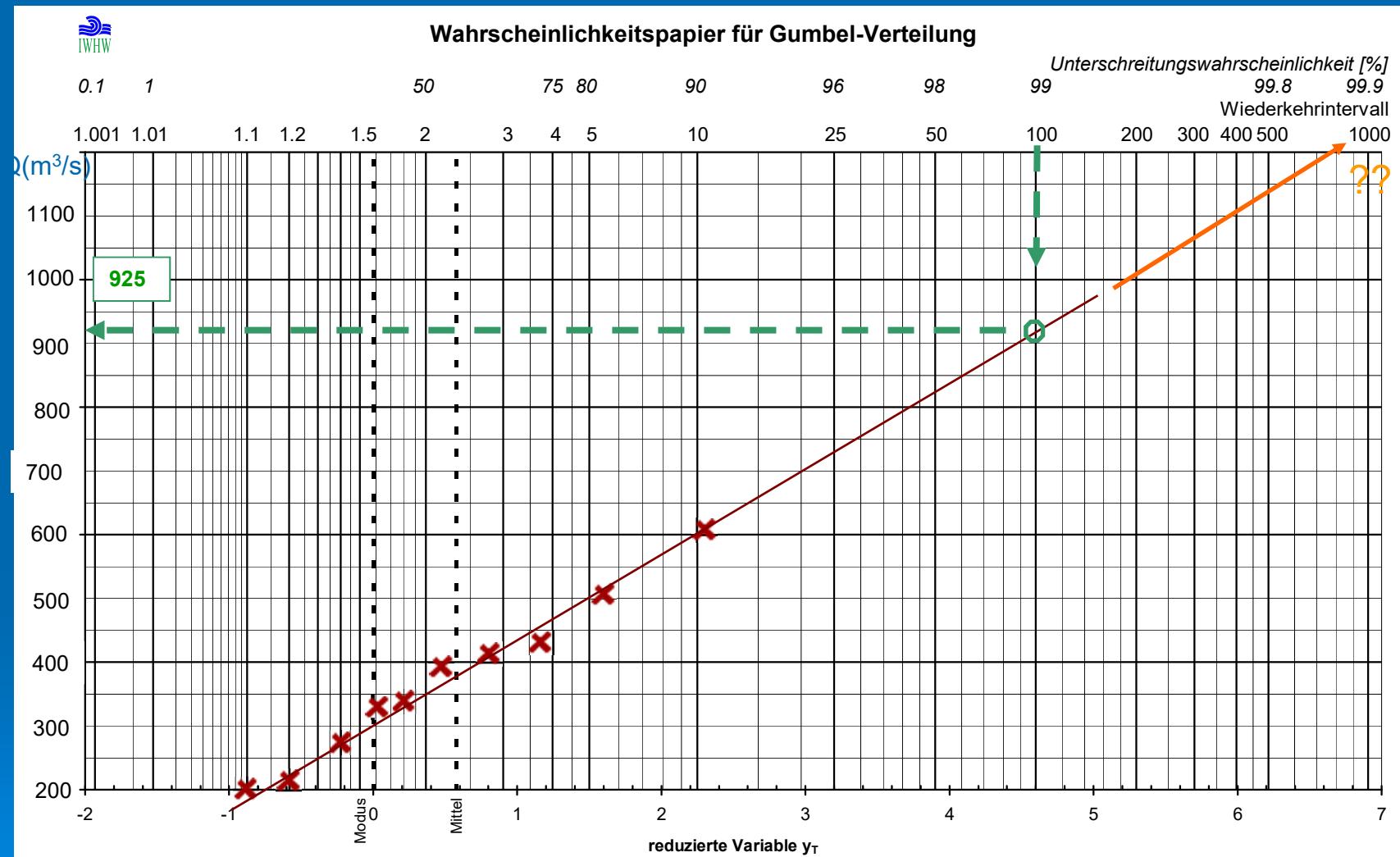
Jahr	Q _{max}	Rank	T	Weib.
1950	342	6	1,7	1,8
1951	415	4	2,5	2,8
1952	199	10	1	1,1
1953	278	8	1,3	1,4
1954	512	2	5	5,5
1955	333	7	1,4	1,6
1956	395	5	2	2,2
1957	607	1	10	11
1958	212	9	1,1	1,2
1959	437	3	3,3	3,7

➤ Plotting Positions

- often Weibull plotting is used

$$F(x) = \frac{k}{n + 1}$$

Graphical Representation



Computational Approach

- Gumbel distribution $F(x)$: has parameters a and c
- Parameters a and c can be related to \bar{x} and s_x

$$F(x_T) = e^{-e^{\frac{-a+\bar{x}T}{c}}} = 1 - \frac{1}{T}$$

$$c = \frac{\pi}{\sqrt{6s_x}} \quad a = \bar{x} - 0,5772 * c$$

Computational Approach

- Gumbel distribution $F(x)$: has parameters a and c
- Parameters a and c can be related to \bar{x} and s_x

$$F(x_T) = e^{-e^{\frac{-a+x_T}{c}}} = 1 - \frac{1}{T}$$

$$c = \frac{\pi}{\sqrt{6s_x}} \quad a = \bar{x} - 0,5772 * c$$

- Estimation of a rare event x_T

$$x_T = \bar{x} + u(T) * s_x \quad \text{Normal distribution}$$

$$x_T = \bar{x} + K(T) * s_x \quad \text{Gumbel distribution}$$

Computational approach

- Estimation of \bar{x} and s_x
- \bar{x} : 373 (m^3/s)
- s_x : 128,3 (m^3/s)
- K_T depends on T and is available in any statistic book
- for $T = 100$ $K_T = 4,323$

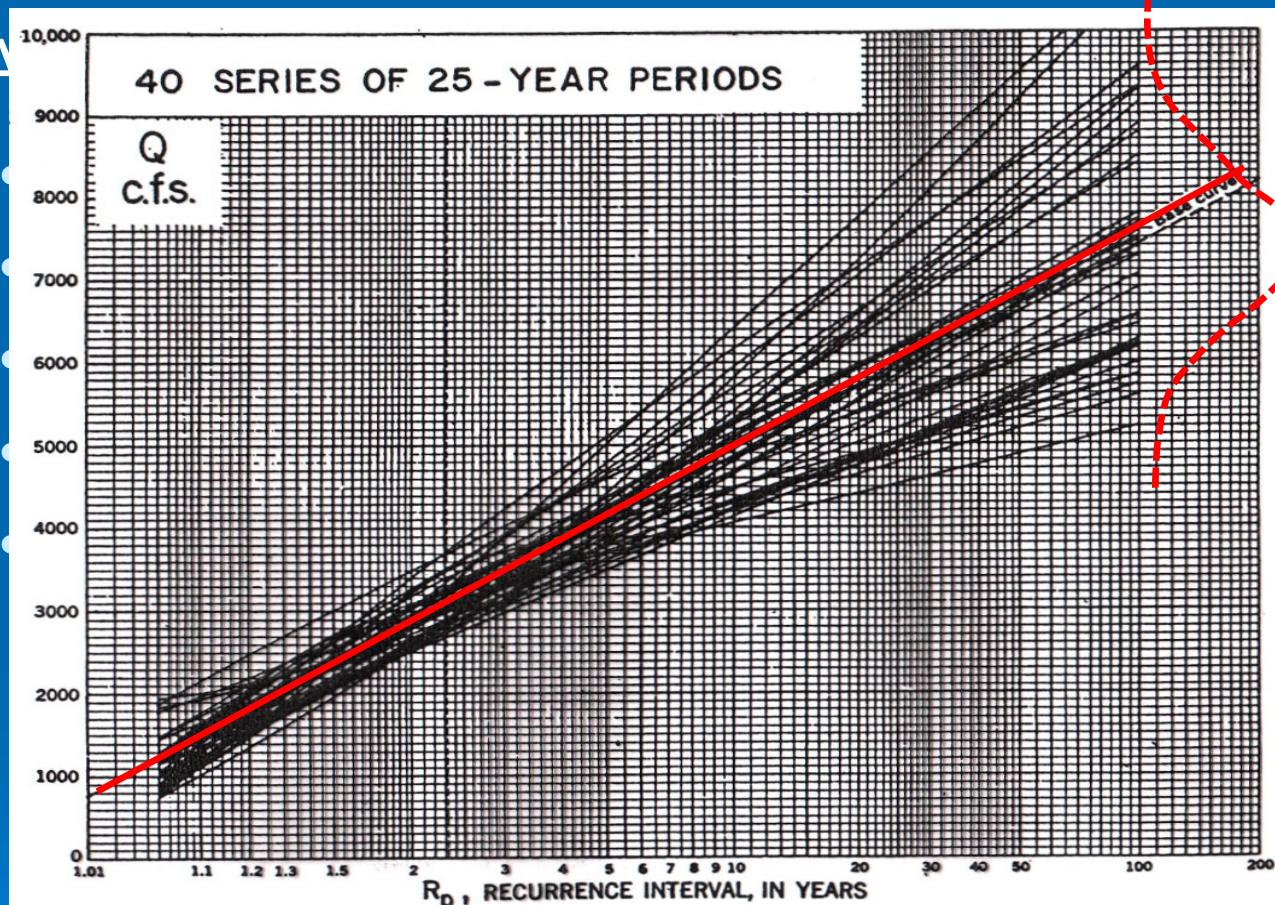
- $x_{T=100} = 928$ (m^3/s)

Sampling uncertainty

- Each estimated value has a pdf (uncertainty)
 - Because we have a small sample
 - Another sample would yield different results
-
- Lets make an experiment!
 - Perfect model and perfect observations
 - 1000 years „observed“ (simulated)
 - Take sub-samples each of 25 years
 - Extreme value analysis for each sub-sample

Sampling uncertainty

➤ A



Using all data

Figure 4.3. Frequency distributions of 40 samples, each of a 25-year size, of the maximum flood discharges on a Gumbel paper graph. Each sample is drawn from a finite population of 1,000 values without replacement. These values follow the double exponential distribution. [According to Benson, 7]

Estimation uncertainty

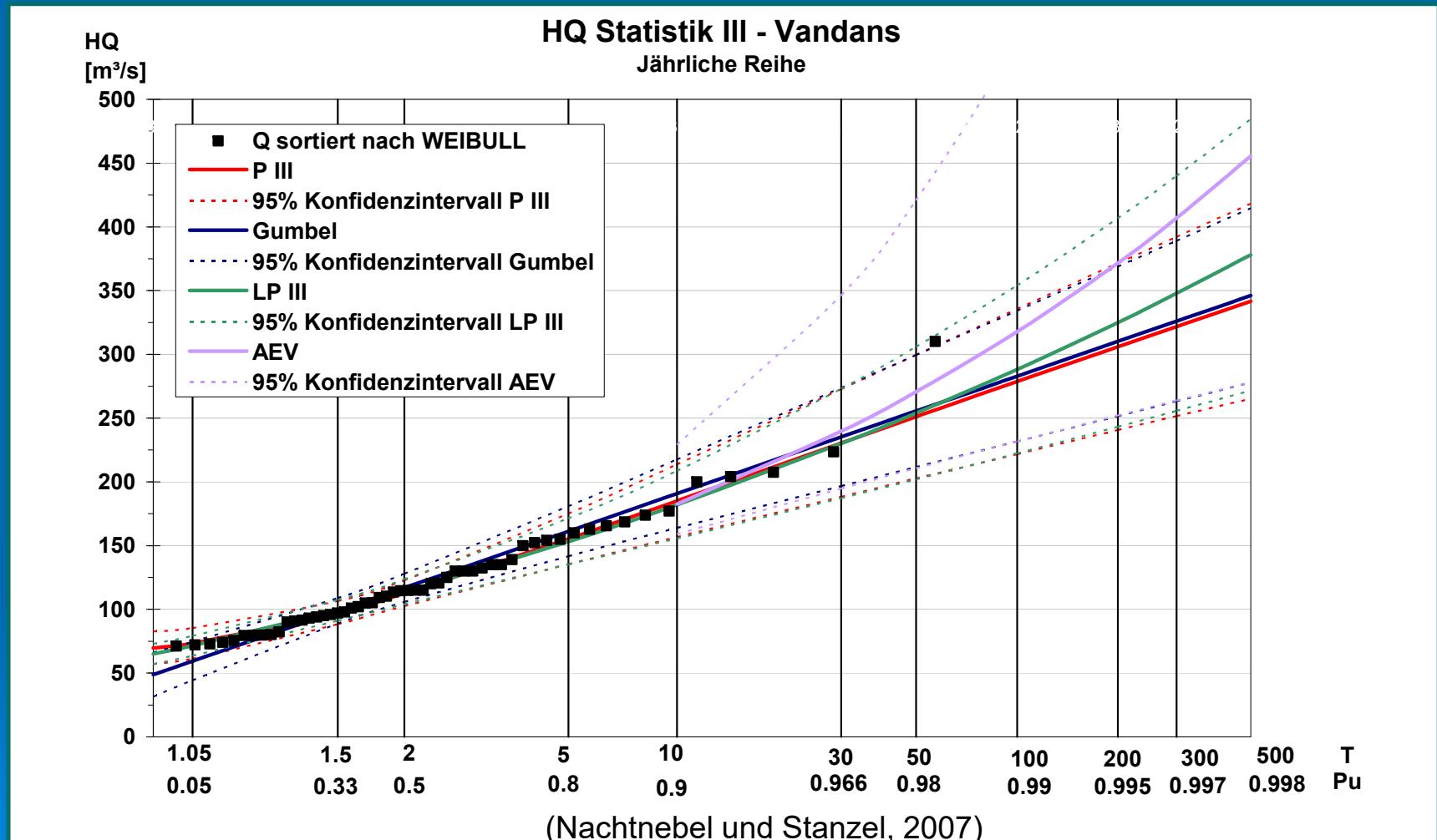
- The shorter the observation length n the larger is the uncertainty
- The larger the variance s_x the larger is the uncertainty
- The lower the probability of occurrence (the larger T) the larger is the uncertainty
- The larger the confidence level α the larger is the uncertainty

$$x_T \pm u(\alpha) * s_T = x_T \pm u(\alpha) * \delta_T \frac{s_x}{\sqrt{n}}$$
$$\delta_T = \sqrt{1+1,14*K_T + 1,1*K_T^2}$$

$$\delta_T = \sqrt{1+1,14*4,323 + 1,1*4,323^2} = 5,146$$

- With probability of 95 % $x_T = 928 \pm 409,18 \text{ (m}^3/\text{s)}$

Comparison of different pdfs (models) fitted to the same data set



Summary and conclusions

- A time series has been observed
- A critical threshold is being defined
- annual or partial series is obtained
- A model is chosen and fitted to the extremes
- Extrapolation and estimation of rare events
(X_{100}, X_{500}, \dots)
- Assessing the uncertainty