Allocation Principles

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Organisation

- Definitions
- Cost allocation in multi-purpose projects
- Allocation of water resources (static approach)
- Dynamic problems
- Summary conclusions

What is Allocation ?

- a resource (water, land, money, time, capacity of education, transport…)
- Several people (user groups, interest groups…) want to have access to it.

What is Allocation ?

- One resource (water, land, money, time, education…)
- Several people (user groups, interest groups…) want to have access to it.

- Who gets how much ?

What is allocation ?

- One resource (water, land, money, time, education…)
- Several people (user groups, interest groups…) want to have access to it.

- Who gets how much ?
- the act of distributing something to designated places or persons
- the assignment or earmarking of something (Business Dict.)

A Multi-purpose Project

- A reservoir is planned to serve
  A) flood protection
  B) Irrigation
  C) hydropower generation

- Several partners (institutions) agree to finance jointly the project

- Problem: Who pays how much?

Methodology

- Specific cost technique
  (alternative justifiable expenditure method)

- Specific costs are the costs which are obviously and uniquely associated to one purpose (A…C)
- Justifiable costs refer to the economically justified present value of costs
- Remaining costs are proportional to the remaining benefits

Specific Cost Technique

Purpose	A	B	C	Total
Individual benefits	0,8	5,8	4,2	10,8
Alternative costs (individual)				
Justified costs				
Specific costs	0,4	1,9	1,8	4,1
Remaining benefits				
Adjoint costs				
Total costs				6,5
Savings (%)				

Specific Cost Technique

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Individual benefits	0,8	5,8	4,2	10,8
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Remaining benefits				
Adjoint costs				2,4
Total costs				6,5
Savings (%)				

Specific Cost Technique

Purpose	A	B	C	Total
Individual benefits	0,8	5,8	4,2	10,8
Alternative costs (individual)	1,0	4,6	3,7	
Justified costs				
Specific costs	0,4	1,9	1,8	4,1
Remaining benefits				
Adjoint costs				2,4
Total costs				6,5
Savings (%)				

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Remaining benefits	0,4	2,7	1,9	5,0
Adjoint costs	0,19	1,3	0,91	2,4
Total costs				6,5
Savings (%)				

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Remaining benefits	0,4	2,7	1,9	5,0
Adjoint costs	0,19	1,3	0,91	2,4
Total costs	0,59	3,2	2,71	6,5
Savings (%)				

Specific Cost Technique

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Total costs	0,59	3,2	2,71	6,5
Savings (%)	8	54	38	100

Specific Cost Technique

- Pro‘s
  4 project alternatives have to be analysed
  three individual projects A,B,C
  one multi-purpose project A+B+C
  simple
- Con‘s
  sum of specific costs is often low because it is difficult to discriminate who needs what

Separable Cost Technique

- Separable costs are those costs which accrue from considering the additional cost by integrating the third purpose

  Sep Cost (C) = Cost (A+B+C) - Cost (A+B)

Separable Cost Technique

Purpose	A	B	C	Total
Individual benefits	0,8	5,8	4,2	10,8
Alternative costs (individual)				
Justified costs				
Separable costs				
Remaining benefits				
Adjoint costs				
Total costs				6,5

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Justified costs	0,8	4,6	3,7	9,1
Separable costs	0,5	2,2	2,5	5,2
Remaining benefits				
Adjoint costs				
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Separable costs	0,5	2,2	2,5	5,2
Remaining benefits	0,3	2,4	1,2	3,9
Adjoint costs	0,1	0,8	0,4	1,3
Total costs				6,5

Separable Cost Technique

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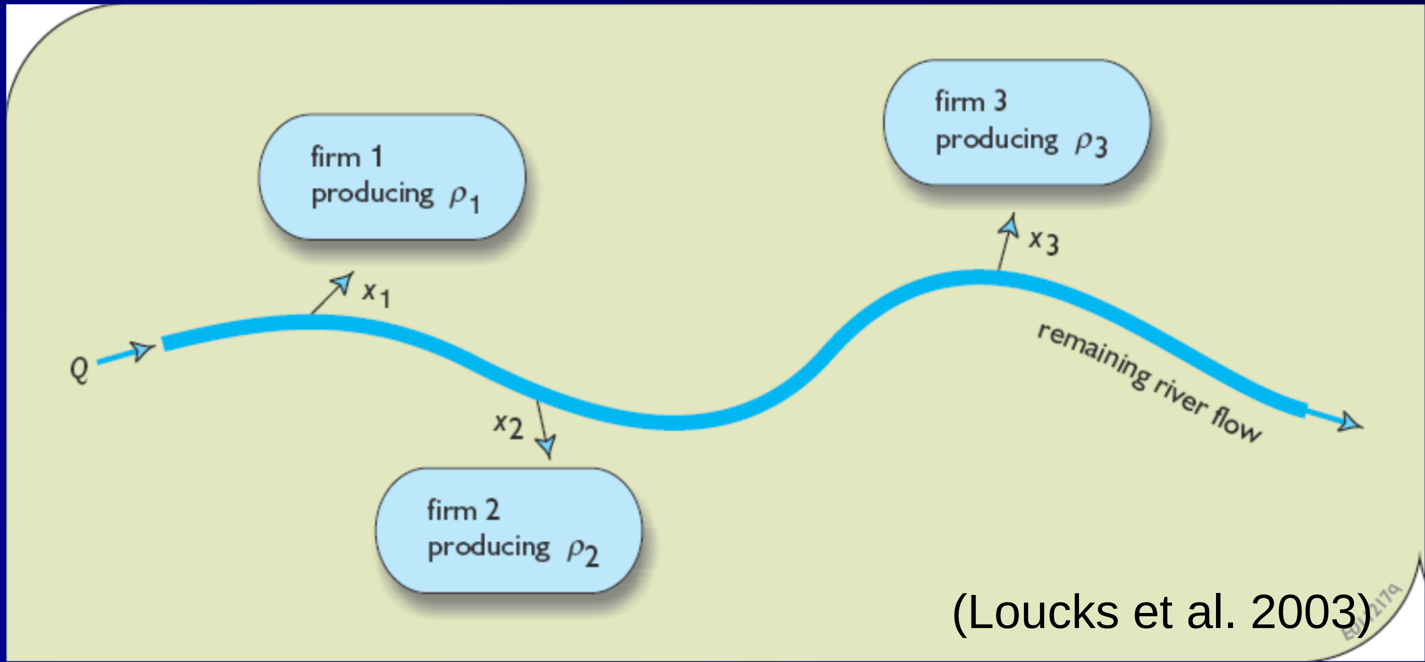
Justified costs	0,8	4,6	3,7	9,1
Separable costs	0,5	2,2	2,5	5,2
Remaining benefits	0,3	2,4	1,2	3,9
Adjoint costs	0,1	0,8	0,4	1,3
Total costs	0,6	3,0	2,9	6,5

Separable Cost Technique

- Pro‘s
  has often a higher discriminative power

- Con‘s
  a higher planning effort
  Alternatives (A+B+C), (A+B), (A+C), (B+C), A,B,C

Allocation of Water Resources
Static Case



(Loucks et al. 2003)

3 farmers utilize the same resource (constant flow Q)
No priority among them (upstream-downstream
For each farm the net benefit function is given

How to allocate the water to obtain MAX(Σ NB_i)

Allocation of Water

Individual benefit functions are given

$$NB_1(x_1) = 6x_1 - x_1^2$$

$$NB_2(x_2) = 7x_2 - 1.5x_2^2$$

$$NB_3(x_3) = 8x_3 - 0.5x_3^2$$

Allocation of Water

Individual benefit functions:

$$NB_1(x_1) = 6x_1 - x_1^2$$

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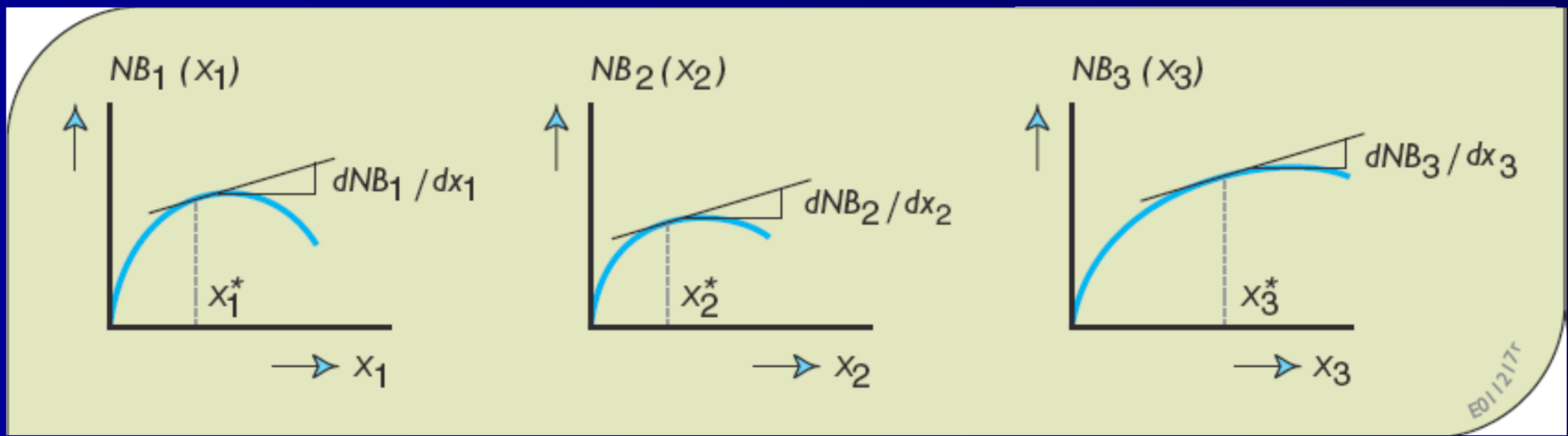

Allocation of Water

Individual benefit functions:

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Constraints:

$x1+x2+x3 <= Q$

$X_i >= 0$

Objective function:

Max $\{\Sigma NB_i + \lambda_0(\Sigma x_i - Q)\}$

See Lagrange approach for optimisation of a contraint problem

Allocation of Water

$$NB_1(x_1) = 6x_1 - x_1^2$$

$$NB_2(x_2) = 7x_2 - 1.5x_2^2$$

$$NB_3(x_3) = 8x_3 - 0.5x_3^2$$

$dNB_1/dx_1 = 6-2x_1$ $x_1=3$ $NB_1 = 9$

$dNB_2/dx_2 = 7-3x_2$ $x_2=7/3$ $NB_2 = 49/6$

$dNB_3/dx_3 = 8-x_3$ $x_3=8$ $NB_3 = 32$

Q=15

All requirements can be satisfied (sum x_i =13 +1/3) and the sum of NB_i is 49+1/6

Allocation of Water

Same problem: but $Q<\Sigma\, x_i$ and all $x_i>=0$

What to do ?

$6-2x_1+\lambda=0$

$7-3x_2+\lambda=0$

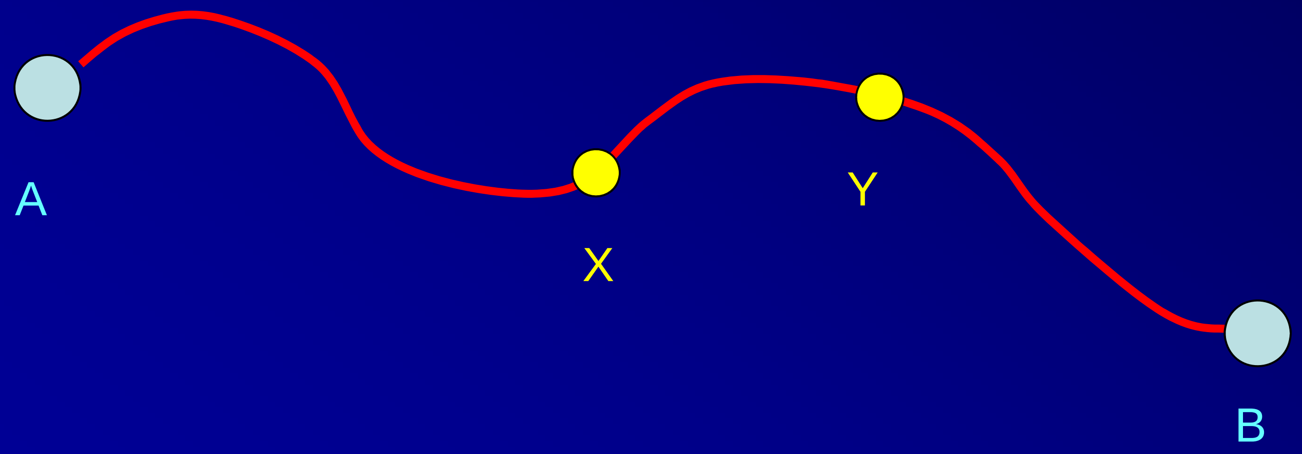
$8-x_3+\lambda=0$

$x_1+x_2+x_3<=Q=10$

$x_1=23/11$ $x_2=19/11$ $x_3=68/11$ $\lambda=-20/11$

Dynamic Programming

- Bellmann principle
  If this trajectory is the shortest way from A to B then it must be also the shortest way from X to Y



In several optimisation problems the optimal solution can be composed of parts of optimal solutions

$Z_j(s_j) = \max_{0 \le x_j \le s_j} \{NB_j(x_j) + Z_{j+1}(s_j - x_j)\}$

Dynamic Programming

Reformulation of the goal function:

$$Z(Q) = \max\{\Sigma NBi(x_i)\} + \lambda \Sigma x_i$$

$$Z(Q) = \max_{0 \le x_1 \le Q}\left\{NB_1(x_1) + \max_{0 \le x_2 \le Q - x_1}\left(NB_2(x_2) + \max_{0 \le x_3 \le Q - x_1 - x_2}(NB_3(x_3))\right)\right\}$$

$Q - x_1 = s_2$

$Q - x_1 - x_2 = s_3$

$$Z_j(s_j) = \max_{0 \le x_j \le s_j}\{NB_j(x_j) + Z_{j+1}(s_j - x_j)\}$$

Dynamic Programming

Several users with different

benefit and

cost function

$$B_j = a_j(1 - \exp(-b_j x_j))$$

$$K_j = c_j x_j^{d_j}$$

constraints

$$\sum x_j = Q \qquad x_j > 0$$

goal function

$$Z(Q) = \max\{\Sigma NBi(x_i)\} + \lambda \Sigma x_i$$

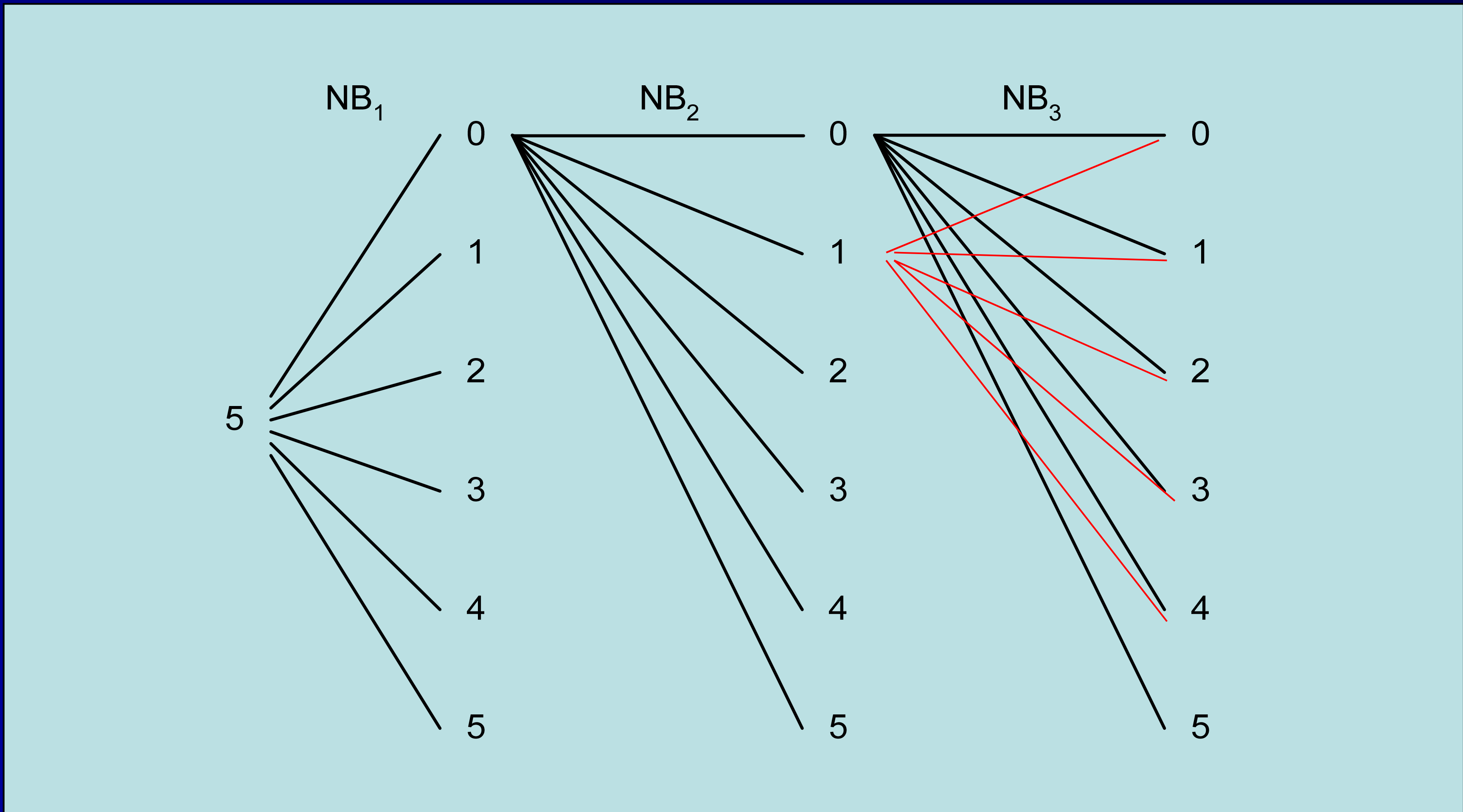
$$Z(Q) = \max_{0 \le x_1 \le Q}\left\{ NB_1(x_1) + \max_{0 \le x_2 \le Q - x_1}\left(NB_2(x_2) + \max_{0 \le x_3 \le Q - x_1 - x_2}\left(NB_3(x_3) \right)\right)\right\}$$

Application of Dynamic Programming

- The coefficients a_i, b_i, c_i, d_i and Q=5 are given
- The steps are assumed to be 1

x_j	$NB_1(x_j)$	$NB_2(x_j)$	$NB_3(x_j)$
0	0	0	0
1	-0,5	6,5	-6,9
2	3,0	10,1	0
3	6,6	10,9	6,3
4	10,0	9,6	11,5
5	13,1	7,0	15,6

A Set of Possible Trajectories



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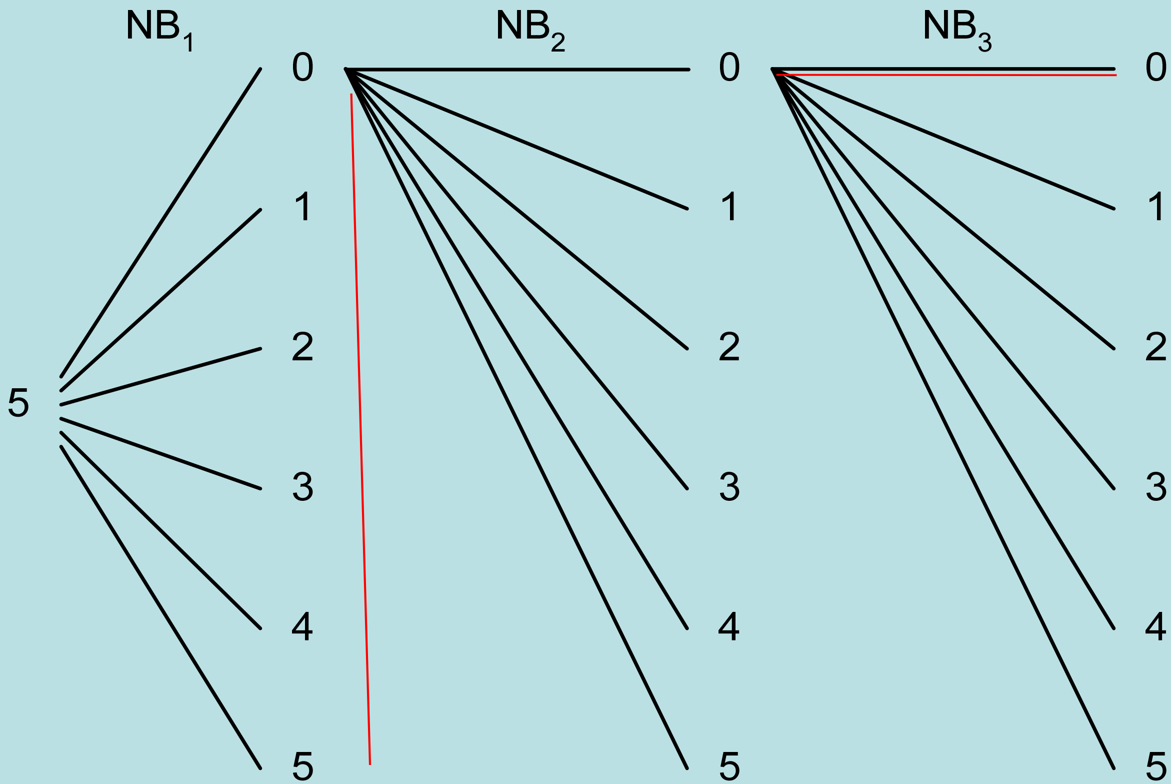

Identifying the Optimal Strategy. Step 1

s_3 the amount which may be available for user 3 and x_3 refers to the options of user 3
x_3^* is the optimal amount given s_3

$NB_3(x_i)$

s_3	x_3	0	1	2	3	4	5	$Z_3(s_3)$	x_3^*
0		0						0	0
1		0	-6,9					0	0
2		0	-6,9	0				0	0
3		0	-6,9	0	6,3			6,3	3
4		0	-6,9	0	6,3	11,5		11,5	4
5		0	-6,9	0	6,3	11,5	15,6	15,6	5

Identifying the Optimal Strategy: Step 2

s_2 the amount which may be available for user 2 + 3 and x_2 refers to the options user 2 has
x_2^* is the optimal amount for user 2 that maximizes $NB_2(x_2^*) + NB_3(s_2-x_2^*)$ given s_2

$NB_2(x_2+Z_3(s_3))$

s_2	x_2	0	1	2	3	4	5	$Z_2(s_2)$	x_2^*
0		0						0	0
1		0	6,5					6,5	1
2		0	6,5	10,1				10,1	2
3		6,3	6,5	10,1	10,9			10,9	3
4		11,5	12,8	10,1	10,9	9,6		12,8	1
5		15,6	18,0	16,4	10,9	9,6	7,0	18,0	1

Identifying the Optimal Strategy: Step 3

s_1 the amount which may be available for user 1+2+3 and x_1 refers to the options of user 1
x_1* is the optimal amount for user 1 that maximizes $NB_1(x_1^*) + NB_2(x_2^*) + NB_3(Q-x_1^*-x_2^*)$ given s_1=Q

$$\mathbf{NB_1(x_1+Z_2(s_2))}$$

Q	x_1	0	1	2	3	4	5	$Z_1(Q)$	x_1^*
s_1=5		18,0	12,3	13,9	16,7	16,5	13,1	18,0	0

Identifying the Optimal Strategy: Step 3

$NB_1(x_1+Z_2(s_2))$

Q	x_1 0	1	2	3	4	5	$Z_1(Q)$	x_1^*
5	18,0	12,3	13,9	16,7	16,5	13,1	18,0	0

The optimal allocation strategy is:

x_1=0

x_2=1

x_3=4

NB= 18,0

Real Life

- Water availability is varying in time
  → complexity increases

- Water input is not known in advance
  → stochastic dynamic programming

- Different interest (flood protection, hydropower,..)
  → multi-objective stochastic dynamic progr.

Summary

- Static and dynamic water allocation problems
- Simplification and analytical approaches
- Discretisation and dynamic programming
- Real world applications are really demanding