Multi-objective Decision Making Techniques

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Structure of Presentation

- Objectives and introduction
- Classification of techniques
- Examples
- Discussion and conclusions
Multi-Objective Decision Making

Objective:
Overview of the concepts applied in MCDM
- One decision maker
  - quantitative (Compromise Programming) and qualitative criteria (ELECTRE I-IV)
a-analysis of pro’s and con’s
  - applicability
Multi-Objective Decision Making

- already in the 1970’s is was recognised that single objective decision making is not successful in natural resources management
- a multi-objective framework including
  - economic
  - social
  - ecological objectives
was developed (Principles and standards in water resources planning USA 1973, 1975; EU-Framework directive on water, 2000)
Example of Objectives, Subobjectives and Indicators

- Improve national economy
  - increase net benefits
    - \( \text{NPV}(\text{€}); \text{multiplier effects, …} \)
  - create job opportunity
    - \( \text{permanent jobs (\#), temporary jobs (\#)} \)…
  - reduce imports
    - \( \text{energy, high tech products,…} \)

………..
Example of Objectives, Subobjectives and Indicators

- Improve environmental conditions
  - preserve/extend aquatic wetlands
    - (area (ha), natural diversity (index)…)
  - preserve/improve groundwater quality
    - (nitrate conc. (mg/l), dissolved iron (mg/l), heavy metals (mg/l), recharge (m3/a))
  - preserve/stabilise endangered species
    - (number (#), reproduction rate (%)…)

.............
Example of Objectives, Subobjectives and Indicators

- Improve the social situation
  - (job opportunities (#/a),
    recreational opportunities (# people/day)......
  - development should be based on regional resources
    - (investment in the region (€),
  - improve equity within society
    - benefits and adverse project impacts should be balanced within the region
  - avoid adverse external impacts
Techniques

- Methods without preferences
- Methods with given preferences
  - distance-based techniques
  - outranking techniques (for discrete alternatives only)
  - value- or utility-based techniques
- Methods with iteratively formulated preferences
General Concept of MO or MCDM

- Environmental Quality
- Economic benefit

Nondominated alternatives
Pareto optimal alternatives
Dominated alternatives

A1
A2
A3
A4
A5
A6
A7
A8
A9
A10

A3 > A5 > A7
A2 > A5 > A7
A6 > A7

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Methods Without Preferences
Identification of the Nondominated Set

1. St. Step
Methods Without Preferences
Identification of the Nondominated Set

1. st step

2nd. step
Methods Without Preferences
Identification of the Nondominated Set

NISE-Approximation (Non Inferior Set Estimation)
Within a few steps a good approximation has been achieved

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Methods Without Preferences

$\varepsilon$-Constraint Method

Stepwise optimisation with a new constraint
Distance Based Techniques

- Require quantitatively expressed criteria
- Require preferences (weights and scales)
- Number of alternatives may be infinite (optimisation)

- Yield a full ranking of alternatives
- Might be iteratively applied
Procedure

- Impact table: expresses the consequences of each alternative with respect to each criterion in measurable units.
- Efficiency or payoff table: transformation of impacts into efficiency measures (scaling).
- Estimation of the overall efficiency ("best solutions").
### Impact Table

<table>
<thead>
<tr>
<th>Criteria</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>Ai</th>
<th>AN</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 (€)</td>
<td>c11</td>
<td>c12</td>
<td>c13</td>
<td>c1i</td>
<td>c1N</td>
</tr>
<tr>
<td>C2   ...</td>
<td>c21</td>
<td>c22</td>
<td>c23</td>
<td>c2i</td>
<td>c2N</td>
</tr>
<tr>
<td>C3 (mg/l)</td>
<td>c31</td>
<td>c32</td>
<td>c33</td>
<td>c3i</td>
<td>c3N</td>
</tr>
<tr>
<td>Cj   ...</td>
<td>cj1</td>
<td>cj2</td>
<td>cj3</td>
<td>cji</td>
<td>cjN</td>
</tr>
<tr>
<td>CJ (ha)</td>
<td>cJ1</td>
<td>cJ2</td>
<td>cJ3</td>
<td>cJi</td>
<td>cJN</td>
</tr>
</tbody>
</table>
Payoff Table

- The physical outcomes have to be transformed into appreciation values (often the efficiency in reaching an objective is used)

![Utility Curve](image)
Payoff Table

- The physical outcomes have to be transformed into appreciation values (often the efficiency in reaching an objective is used).

\[ a_{i,j} = \frac{c_{i,j} - c_{j\text{Min}}}{c_{j\text{Max}} - c_{j\text{Min}}} \]
Sometimes Utility and Membership Functions are Used

- Increase in water temperature
- Dissolved Oxygen
- Water depth in river
- Available water volume
- Variability in width
### Efficiency or Payoff Table

<table>
<thead>
<tr>
<th>Criteria</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_i )</th>
<th>( A_N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 ) (%)</td>
<td>( a_{11} )</td>
<td>( a_{12} )</td>
<td>( a_{13} )</td>
<td>( a_{1i} )</td>
<td>( a_{1N} )</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>( a_{21} )</td>
<td>( a_{22} )</td>
<td>( a_{23} )</td>
<td>( a_{2i} )</td>
<td>( a_{2N} )</td>
</tr>
<tr>
<td>( C_3 ) (%)</td>
<td>( a_{31} )</td>
<td>( a_{32} )</td>
<td>( a_{33} )</td>
<td>( a_{3i} )</td>
<td>( a_{3N} )</td>
</tr>
<tr>
<td>( C_j )</td>
<td>( a_{j1} )</td>
<td>( a_{j2} )</td>
<td>( a_{j3} )</td>
<td>( a_{ji} )</td>
<td>( a_{jN} )</td>
</tr>
<tr>
<td>( C_J ) (%)</td>
<td>( a_{J1} )</td>
<td>( a_{J2} )</td>
<td>( a_{J3} )</td>
<td>( a_{Ji} )</td>
<td>( a_{JN} )</td>
</tr>
</tbody>
</table>

*full set of alternatives \( A \)*
Selection of Favourable Solutions

Compromise Programming

Diagram:
- Economic Objective \( O_1(S) \)
- Ecological Objective
- Ideal Point
- Distance: \( D_i \) unscaled, \( d_i \) scaled
- Point A
Scaled Representation

Economic Objective $O_1$

Ecological Objective $O_2$

Ideal point

$A_i$

$d_{i,1}$

$d_{i,2}$

$a_{i,1}$

$a_{i,2}$
Identification of Favourable Solutions

\[ d_{i,j} = 1 - a_{i,j} \]

\[ L_i(p) = \sum(w_j \cdot d_{i,j}^p)^{1/p} \]

**What is the role of weights?**
**What is the role of p?**
Ranking of Alternatives

- For a set of given weights and a $p$ value the distance of each alternative from the ideal point can be computed.
- The alternatives are ranked in increasing distances.
- The alternatives with small distances are preferred.
Outranking Techniques

- Often, a pairwise comparison of alternatives is performed. e.g. A3>A4, A5>A4, A4>A2, A3>A2

In ELECTRE (I) only an incomplete ranking can be achieved.
In ELECTRE (IV) a complete ranking is achieved.

Both approaches require weights and scales for describing the preferences.
**ELECTRE I**

- Simple example: 2 alternatives, 3 criteria

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>W</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>c11</td>
<td>c12</td>
<td>w1</td>
<td>s1</td>
</tr>
<tr>
<td>C2</td>
<td>c21</td>
<td>c22</td>
<td>w2</td>
<td>s2</td>
</tr>
<tr>
<td>C3</td>
<td>c31</td>
<td>c32</td>
<td>w3</td>
<td>s3</td>
</tr>
</tbody>
</table>
ELECTRE I

C1 NPV in (Mio €)
C2 (mg/l) water pollution
C3 (# of created jobs)
ELECTRE I

- Impact table

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>W</th>
<th>Scale</th>
<th>Best</th>
<th>Worst</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1.5</td>
<td>1.8</td>
<td>0.5</td>
<td>10</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>C2</td>
<td>10</td>
<td>20</td>
<td>0.2</td>
<td>10</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>C3</td>
<td>100</td>
<td>120</td>
<td>0.3</td>
<td>10</td>
<td>200</td>
<td>0</td>
</tr>
</tbody>
</table>

W weight
S scale
ELECTRE: Concordance and Discordance

- Concordance expresses the dominance of $A_i > A_j$
- Discordance expresses the weakness of $A_i < A_j$

\[
CO(i, j) = \left( \sum_k w_k \left( \text{for } c(i, k) > c(j, k) \right) + \frac{1}{2} \sum_k w_k \left( \text{for } c(i, k) = c(j, k) \right) \right) / \sum_k w_k
\]

\[
DI(i, j) = \text{Max} \left\{ \frac{\text{scale} \left( c(i, k) - c(j, k) \right)}{\text{Scale}(k)} \right\}
\]
ELECTRE I

- \( CO(1,2) = 0.2, \ CO(2,1) = 0.8 \)
- \( DI(1,2) = \text{Max}(0.3, 0.2) = 0.3 \)
- \( DI(2,1) = \text{Max}(10/50) = 0.2 \)
- Finally two matrices \( CO(,) \) and \( DI(,) \) are obtained
- A threshold level \( CO^* \) and \( DI^* \) is introduced (e.g. \( CO^* = 0.75, DI^* = 0.2 \) then \( A2 > A1 \)
Other Distance Based Techniques

Goal Programming

Compromise Programming

Cooperative Game Theory
Utility Based Techniques

- Often single attribute utility theory is applied
- If possible, MAUT (Multi-Attribute-Utility Theory) should be applied

![Diagram showing utility value vs. available amount of water (Cj in m3)]
Utility Theory

(1) Impact matrix
(2) Transformation of impacts into utilities
(3) Definition of weights for each criterion
(4) Overall utility value $UV_i$ of alternative $A_i$ is

$$UV_i = \sum_j w_j \cdot uv_{ij}$$
1-D and 2-D Utility Functions

1-D Utility

2-D Utility

Utility

1

0

Xmin

Xmax

Criterion X

u

u(x) + u(c)

x

x1

x2

c1

c2

d1

d2

u(x) + u(c)

u(x) + u(c)

u(x) + u(c)
Conclusions

- Numerous methods exist for evaluation of water resources projects
- Multi-objective decision making is a daily problem
- The concepts of multiple objectives is found in many international/national case studies
- The major steps are in the problem definition in the impact assessment in knowing about the preferences
- The numerical methods are helpful in improving the understanding of the problem and for exploring the feasible domain