# Allocation Principles

H.P. Nachtnebel

# Organisation

- Definitions
- Cost allocation in multi-purpose projects
- Allocation of water resources (static approach)
- Dynamic problems
- Summary conclusions

# What is allocation ?

- One resource (water, land, money, time, capacity of education, transport…)
- Several people (user groups, interest groups…) want to have access to it.

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- One resource (water, land, money, time, education…)
- Several people (user groups, interest groups…) want to have access to it.

- Who gets how much ?
- the act of distributing something to designated places or persons
- the assignment or earmarking of something (Business Dict.)

# A Multi-purpose Project

- A reservoir is planned to serve
  A) flood protection
  B) Irrigation
  C) hydropower generation

- Several partners (institutions) agree to finance jointly the project

- Problem: Who pays how much?

# Methodology

- Specific cost technique
  (alternative justifiable expenditure method)

- Specific costs are the costs which are obviously and uniquely associated to one purpose (A…C)
- Justifiable costs refer to the economically justified present value of costs
- Remaining costs are proportional to the remaining benefits

# Specific Cost Technique

| Purpose | A | B | C | Total |
|---|---|---|---|---|
| Individual benefits | 0,8 | 5,8 | 4,2 | 10,8 |
| Alternative costs (individual) | | | | |
| Justified costs | | | | |
| Specific costs | 0,4 | 1,9 | 1,8 | 4,1 |
| Remaining benefits | | | | |
| Adjoint costs | | | | |
| Total costs | | | | 6,5 |
| Savings (%) | | | | |

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| Remaining benefits | 0,4 | 2,7 | 1,9 | 5,0 |
| Adjoint costs | 0,19 | 1,3 | 0,91 | 2,4 |
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| Total costs | 0,59 | 3,2 | 2,71 | 6,5 |
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| Savings (%) | 8 | 54 | 38 | 100 |

# Specific Cost Technique

- Pro‘s
  4 project alternatives have to be analysed
  three individual projects A,B,C
  one multi-purpose project A+B+C
  simple
- Con‘s
  sum of specific costs is often low because it is difficult to discriminate who needs what

# Separable Cost Technique

- Separable costs are those costs which accrue from considering the additional cost by integrating the third purpose

  Sep Cost (C) = Cost (A+B+C) - Cost (A+B)

# Separable Cost Technique

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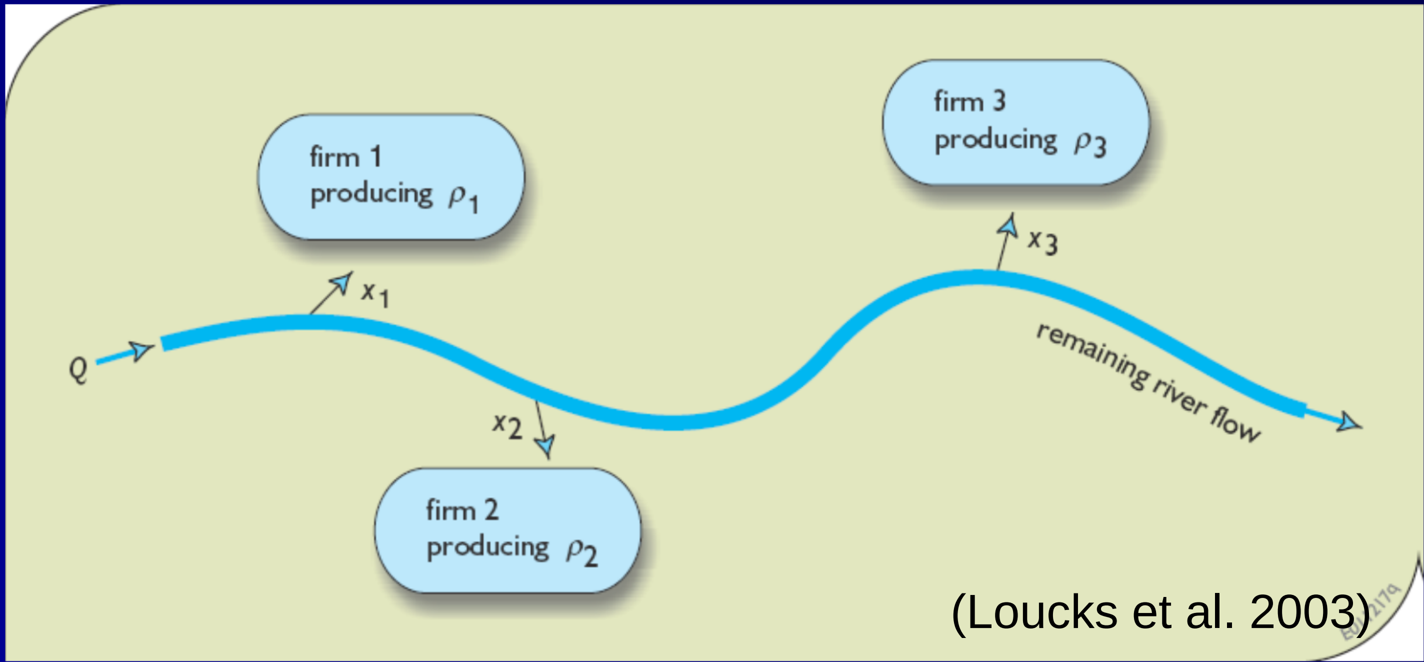
| Remaining benefits | 0,3 | 2,4 | 1,2 | 3,9 |
| Adjoint costs | 0,1 | 0,8 | 0,4 | 1,3 |
| Total costs | 0,6 | 3,0 | 2,9 | 6,5 |

# Separable Cost Technique

- Pro‘s
  has often a higher discriminative power

- Con‘s
  a higher planning effort
  Alternatives (A+B+C), (A+B), (A+C), (B+C), A,B,C

# Allocation of Water Resources
# Static Case




(Loucks et al. 2003)

3 farmers utilize the same resource (constant flow Q)
No priority among them (upstream-downstream
For each farm the net benefit function is given

How to allocate the water to obtain MAX($\Sigma$ $NB_i$)

# Allocation of Water

Individual benefit functions

$$NB_1(x_1) = 6x_1 - x_1^2$$

$$NB_2(x_2) = 7x_2 - 1.5x_2^2$$

$$NB_3(x_3) = 8x_3 - 0.5x_3^2$$

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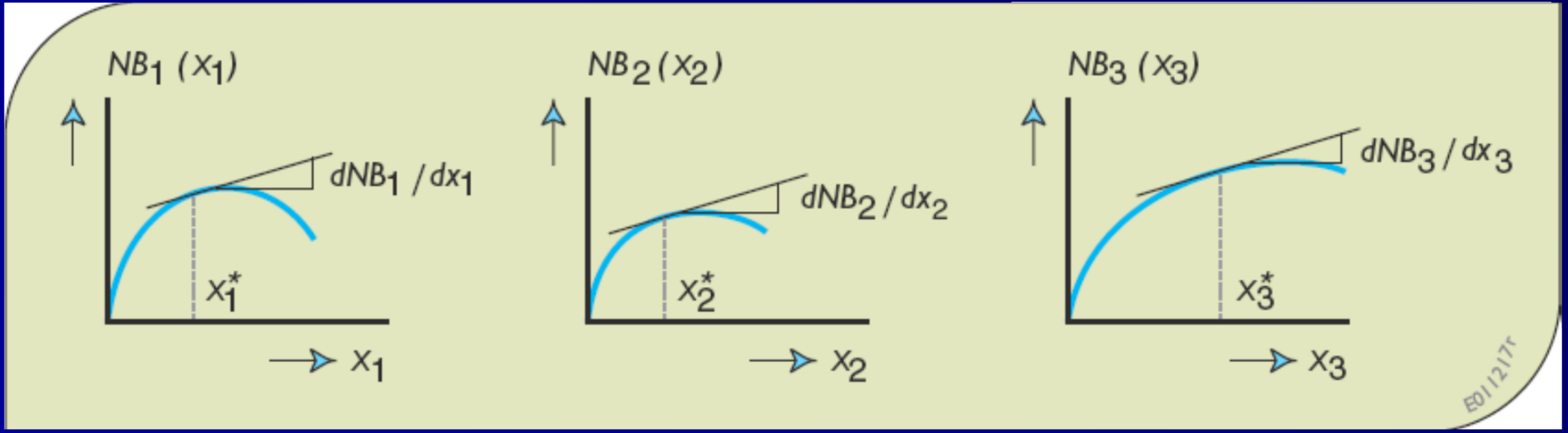

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Constraints:

x1+x2+x3<= Q

$X_i$>=0

Objective function:

Max ($\Sigma NB_i + \lambda_0 \Sigma x_i$)

# Allocation of Water

$$NB_1(x_1) = 6x_1 - x_1^2$$

$$NB_2(x_2) = 7x_2 - 1.5x_2^2$$

$$NB_3(x_3) = 8x_3 - 0.5x_3^2$$

| | | |
|---|---|---|
| $dNB_1/dx_1 = 6-2x_1$ | $x_1=3$ | $NB_1 = 9$ |
| $dNB_2/dx_2 = 7-3x_2$ | $x_2=7/3$ | $NB_2 = 49/6$ |
| $dNB_3/dx_3 = 8-x_3$ | $x_3=8$ | $NB_3 = 32$ |

Q=15

All requirements can be satisfied (sum $x_i$ =13 +1/3) and the sum of $NB_i$ is 49+1/6

# Allocation of Water

Same problem: but $Q < \Sigma x_i$ and all $x_i >= 0$

What to do ?

$6-2x_1+\lambda = 0$

$7-3x_2+\lambda=0$

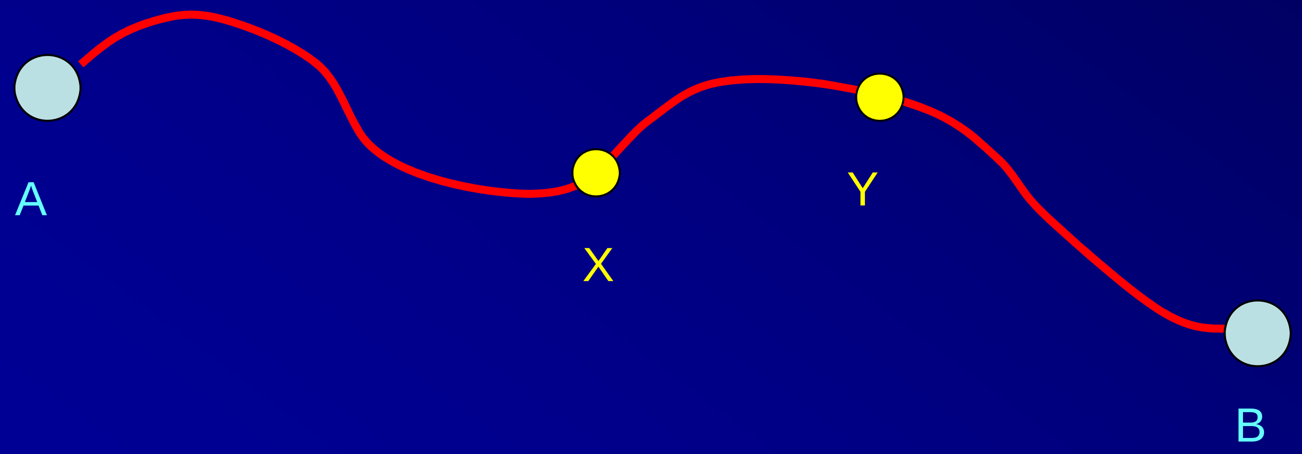
$8-x_3+\lambda=0$

$x_1+x_2+x_3<=Q=10$

$x_1=23/11$ $x_2=19/11$ $x_3=68/11$ $\lambda=-20/11$

# Dynamic Programming

- Bellmann principle
  If this trajectory is the shortest way from A to B then it must be also the shortest way from X to Y




In several optimisation problems the optimal solution can be composed of parts of optimal solutions

$Z_j(s_j) = \max_{0 \le x_j \le s_j} \{NB_j(x_j) + Z_{j+1}(s_j - x_j)\}$

# Dynamic Programming

Reformulation of the goal function:

$$Z(Q) = \max\{\Sigma NBi(x_i)\} + \lambda \Sigma x_i$$

$$Z(Q) = \max_{0 \le x_1 \le Q} \left\{ NB_1(x_1) + \max_{0 \le x_2 \le Q - x_1} \left( NB_2(x_2) + \max_{0 \le x_3 \le Q - x_1 - x_2} \left( NB_3(x_3) \right) \right) \right\}$$

$Q-x_1=s_2$
$Q-x_1-x_2=s_3$

$$Z_j(s_j) = \max_{0 \le x_j \le s_j} \{NB_j(x_j) + Z_{j+1}(s_j - x_j)\}$$

# Dynamic Programming

Several users with different

benefit and

cost function

$$B_j = a_j(1 - \exp(-b_j x_j))$$

$$K_j = c_j x_j^{d_j}$$

constraints

$$\sum x_j = Q \qquad x_j > 0$$

goal function

$$Z(Q) = \max\{\Sigma NBi(x_i)\} + \lambda \Sigma x_i$$

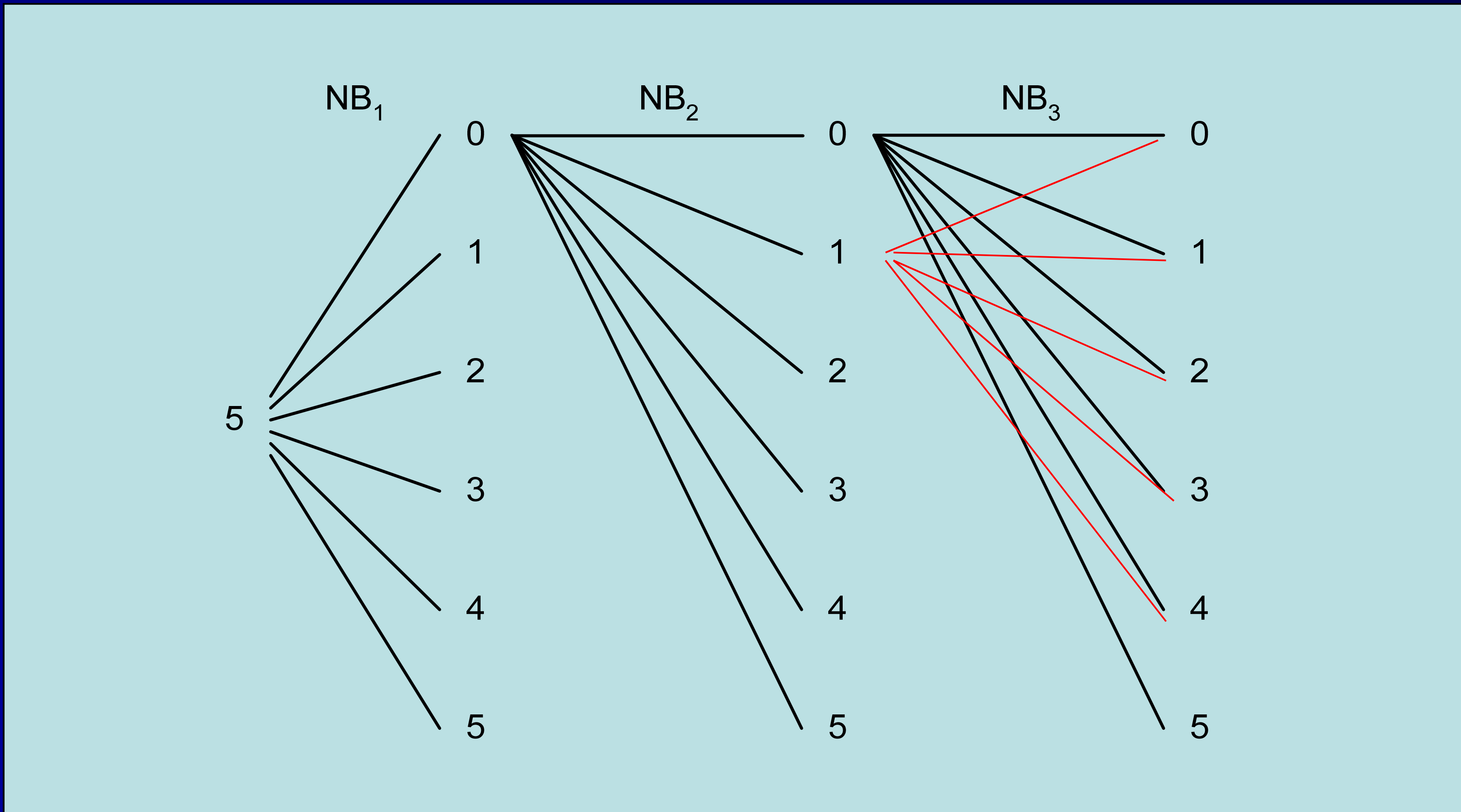
$$Z(Q) = \max_{0 \le x_1 \le Q} \left\{ NB_1(x_1) + \max_{0 \le x_2 \le Q - x_1} \left( NB_2(x_2) + \max_{0 \le x_3 \le Q - x_1 - x_2} \left( NB_3(x_3) \right) \right) \right\}$$

# Application of Dynamic Programming

- The coefficients $a_i,b_i,c_i,d_i$ and Q=5 are given
- The steps are assumed to be 1

| $x_j$ | $NB_1(x_j)$ | $NB_2(x_j)$ | $NB_3(x_j)$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | -0,5 | 6,5 | -6,9 |
| 2 | 3,0 | 10,1 | 0 |
| 3 | 6,6 | 10,9 | 6,3 |
| 4 | 10,0 | 9,6 | 11,5 |
| 5 | 13,1 | 7,0 | 15,6 |

# A Set of Possible Trajectories



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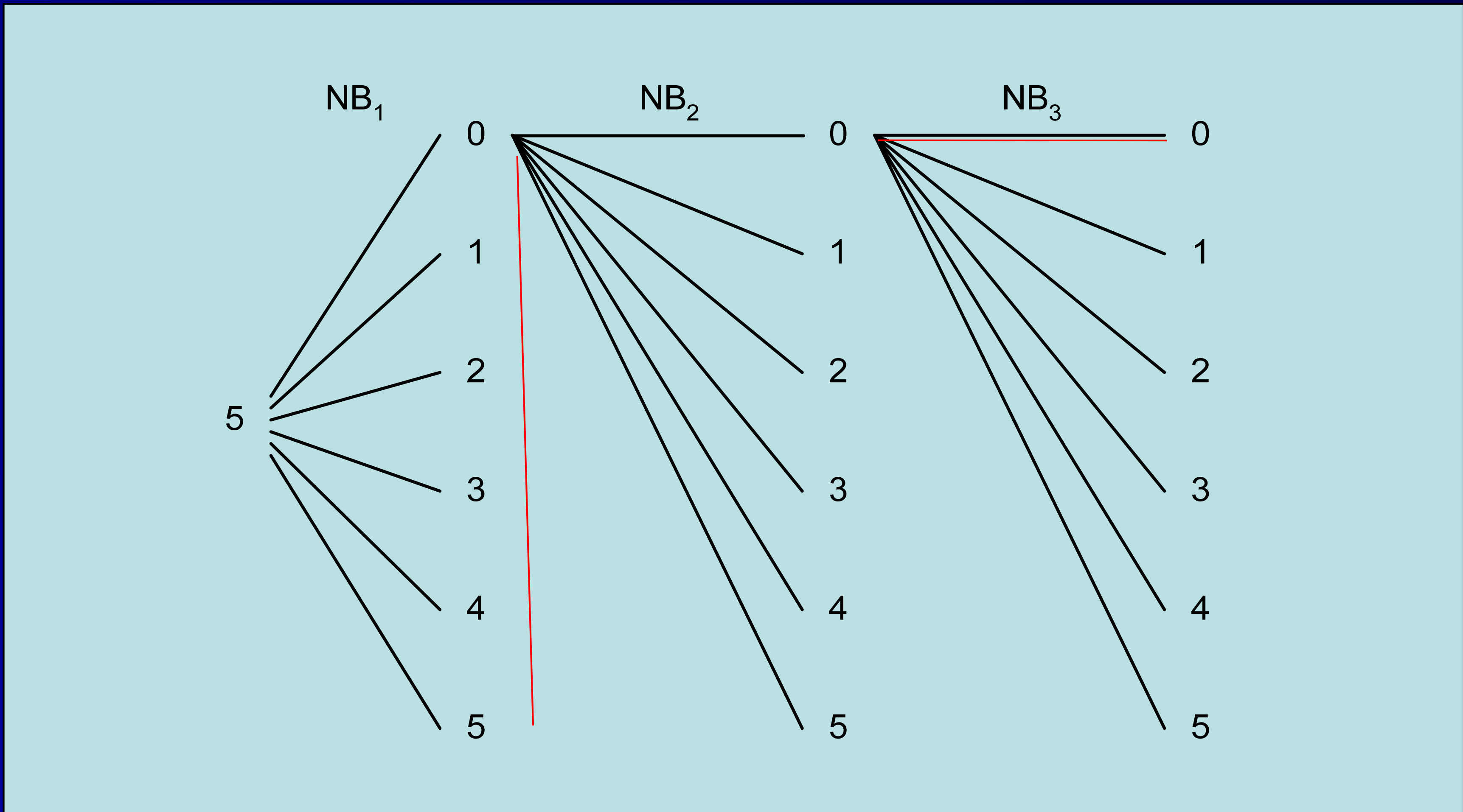

# Identifying the Optimal Strategy. Step 1

**$NB_3(x_i)$**

| $s_3$ | $x_3$ | 0 | 1 | 2 | 3 | 4 | 5 | $Z_3(s_3)$ | $x_3^*$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | | 0 | | | | | | 0 | 0 |
| 1 | | 0 | -6,9 | | | | | 0 | 0 |
| 2 | | 0 | -6,9 | 0 | | | | 0 | 0;2 |
| 3 | | 0 | -6,9 | 0 | 6,3 | | | 6,3 | 3 |
| 4 | | 0 | -6,9 | 0 | 6,3 | 11,5 | | 11,5 | 4 |
| 5 | | 0 | -6,9 | 0 | 6,3 | 11,5 | 15,6 | 15,6 | 5 |

# Identifying the Optimal Strategy: Step 2

**$NB_2(x_2+Z_3(s_3))$**

| $s_2$ | $x_2$ | 0 | 1 | 2 | 3 | 4 | 5 | $Z_2(s_2)$ | $x_2$* |
|---|---|---|---|---|---|---|---|---|---|
| 0 | | 0 | | | | | | 0 | 0 |
| 1 | | 0 | 6,5 | | | | | 6,5 | 1 |
| 2 | | 0 | 6,5 | 10,1 | | | | 10,1 | 2 |
| 3 | | 6,3 | 6,5 | 10,1 | 10,9 | | | 10,9 | 3 |
| 4 | | 11,5 | 12,8 | 10,1 | 10,9 | 9,6 | | 12,8 | 1 |
| 5 | | 15,6 | 18,0 | 16,4 | 10,9 | 9,6 | 7,0 | 18,0 | 1 |

# Identifying the Optimal Strategy: Step 3

$$NB_1(x_1+Z_2(s_2))$$

| Q | $x_1$ | 0 | 1 | 2 | 3 | 4 | 5 | $Z_1(Q)$ | $x_1^*$ |
|---|---|---|---|---|---|---|---|---|---|
| 5 | | 18,0 | 12,3 | 13,9 | 16,7 | 16,5 | 13,1 | 18,0 | 0 |

# Identifying the Optimal Strategy: Step 3

**$NB_1(x_1+Z_2(s_2))$**

| Q | $x_1$ | 0 | 1 | 2 | 3 | 4 | 5 | $Z_1(Q)$ | $x_1^*$ |
|---|---|---|---|---|---|---|---|---|---|
| 5 | | 18,0 | 12,3 | 13,9 | 16,7 | 16,5 | 13,1 | 18,0 | 0 |

The optimal allocation strategy is:

$x_1=0$

$x_2=1$

$x_3=4$

NB= 18,0

# Real Life

- Water availability is varying in time
  → complexity increases

- Water input is not known in advance
  → stochastic dynamic programming

- Different interest (flood protection, hydropower,..)
  → multi-objective stochastic dynamic progr.

# Summary

- Static and dynamic water allocation problems
- Simplification and analytical approaches
- Discretisation and dynamic programming
- Real world applications are really demanding